2020.1.16
物性物理学C

反応拡散系とパターン形成

Turingパターン（静止パターン）

$$\frac{\partial u}{\partial t} = -u^3 + u - 4v + D_u \nabla^2 u$$

$$\frac{\partial v}{\partial t} = u - 3v - a + D_v \nabla^2 v$$

$a = 0$では$u = v = 0$が固定点

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix}$$

$\mathrm{tr}\,\mathrm{A} = -2$、$\det \mathrm{A} = 1$より安定

拡散方程式

$$\frac{\partial u}{\partial t} = D\nabla^2 u$$

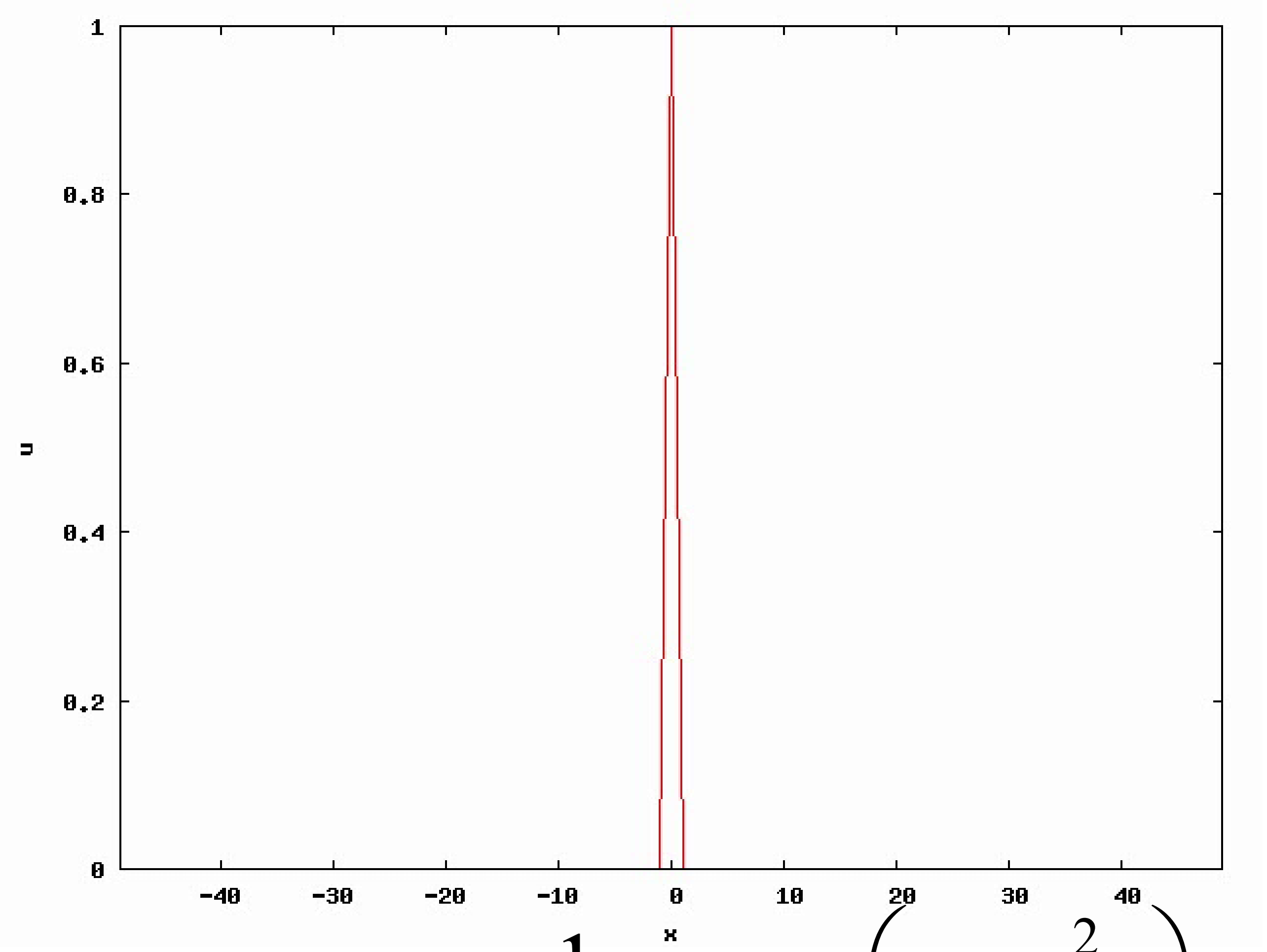
1次元の場合

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$$

初期値:

$$u(x, t = 0) = \delta(x)$$

1次元で初期値がδ関数の解　：　Green関数



$$u(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right)$$

$$u = 0 + \int \mathrm{d}k \ \Delta u(k) e^{ikx}$$

$$v = 0 + \int \mathrm{d}k \ \Delta v(k) e^{ikx}$$ とおいて

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = \begin{pmatrix} 1 - D_u k^2 & -4 \\ 1 & -3 - D_u k^2 \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix}$$

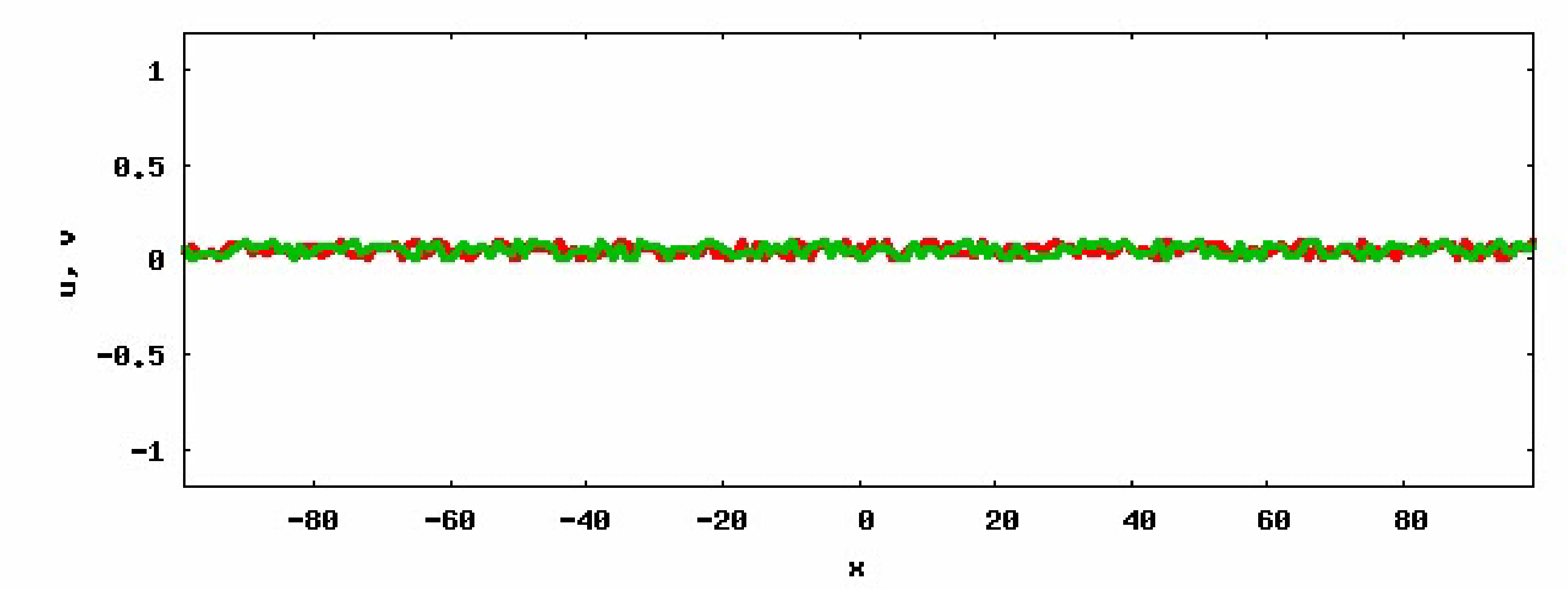
不安定化する条件は

$$D_v > 9 D_u$$

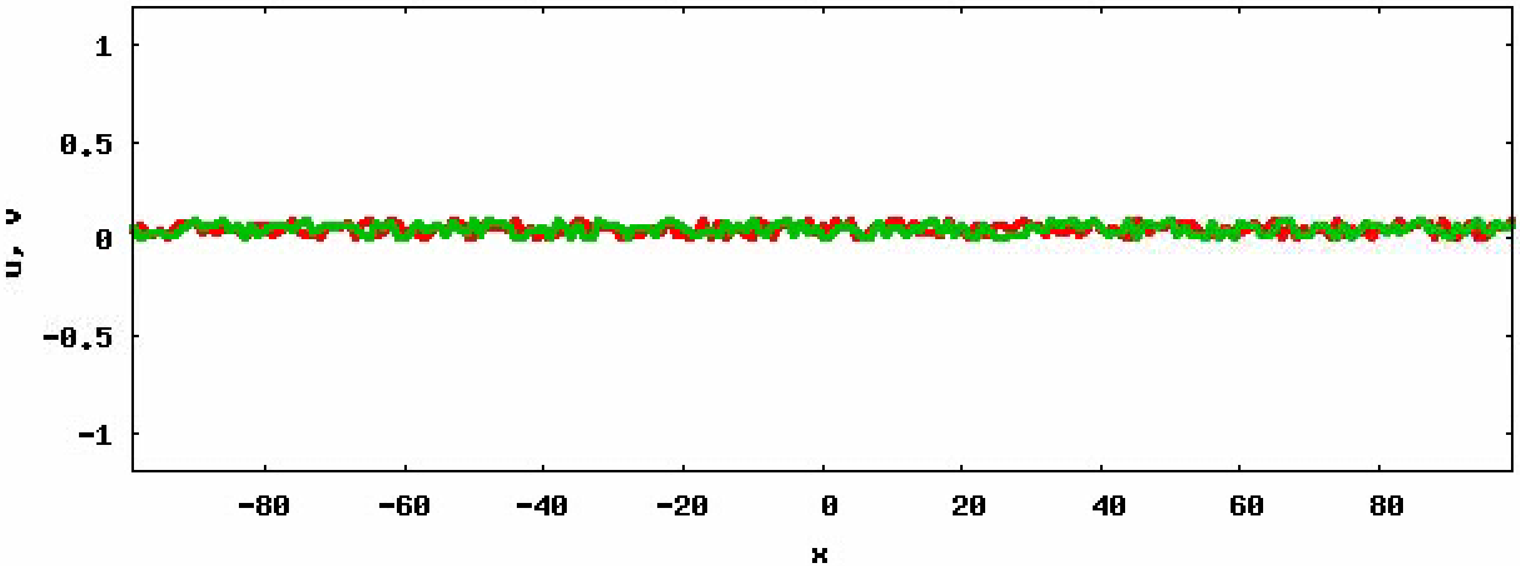
はじめに不安定化する波数は $k = \sqrt{\frac{1}{2D_u} - \frac{3}{2D_v}}$

1次元では

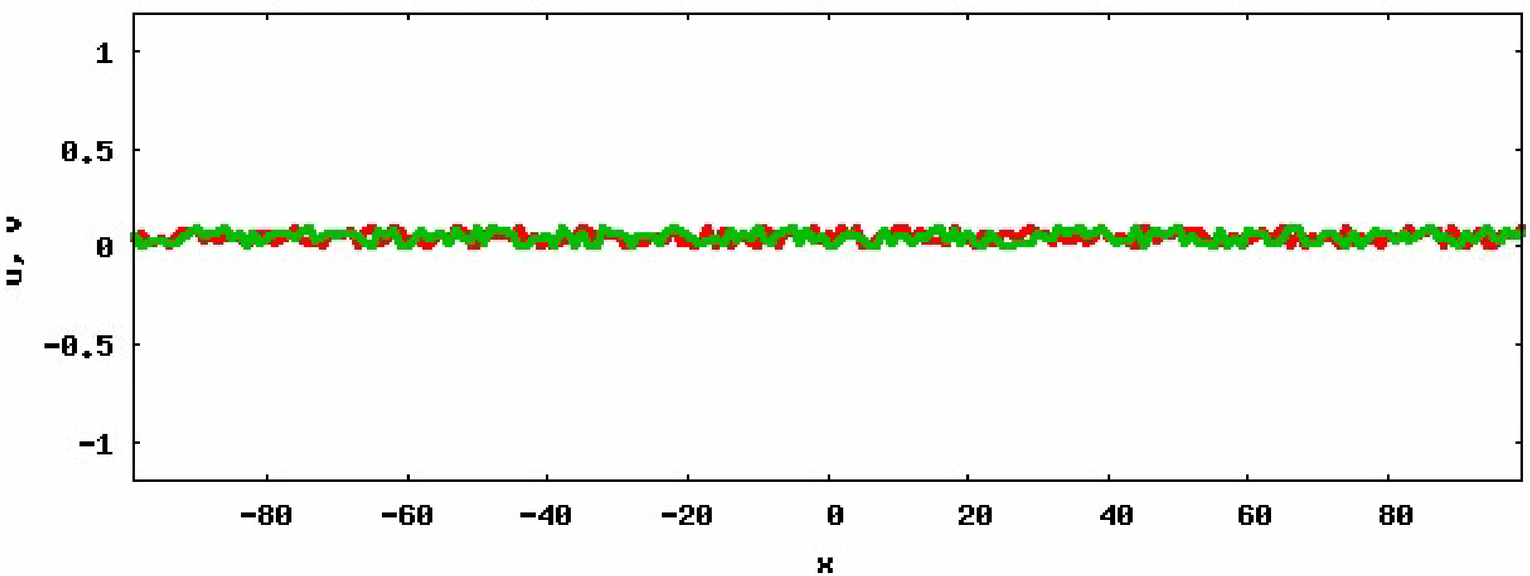
$$a = 0, \quad D_u = 1, D_v = 5$$



$a = 0, \quad D_u = 1, D_v = 20$

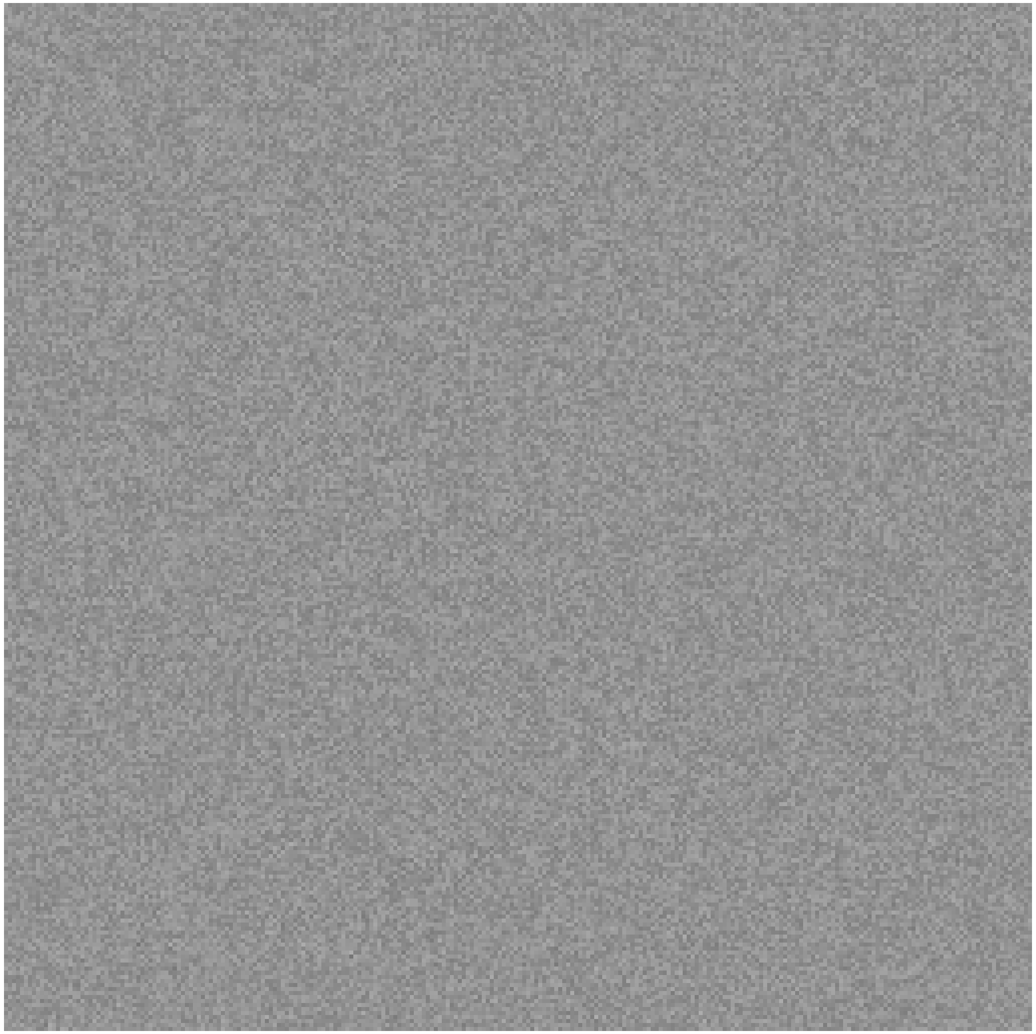


$a = 0, \quad D_u = 2, D_v = 20$

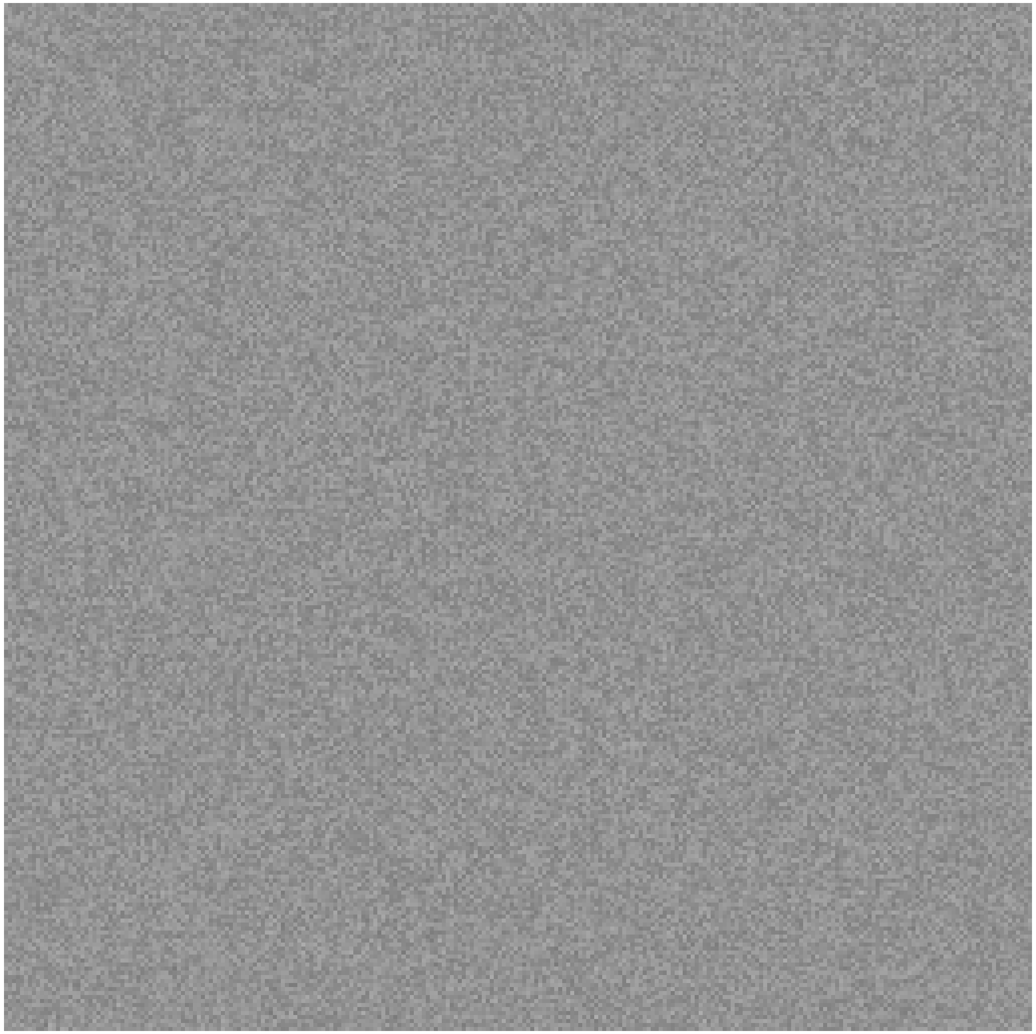


2次元では

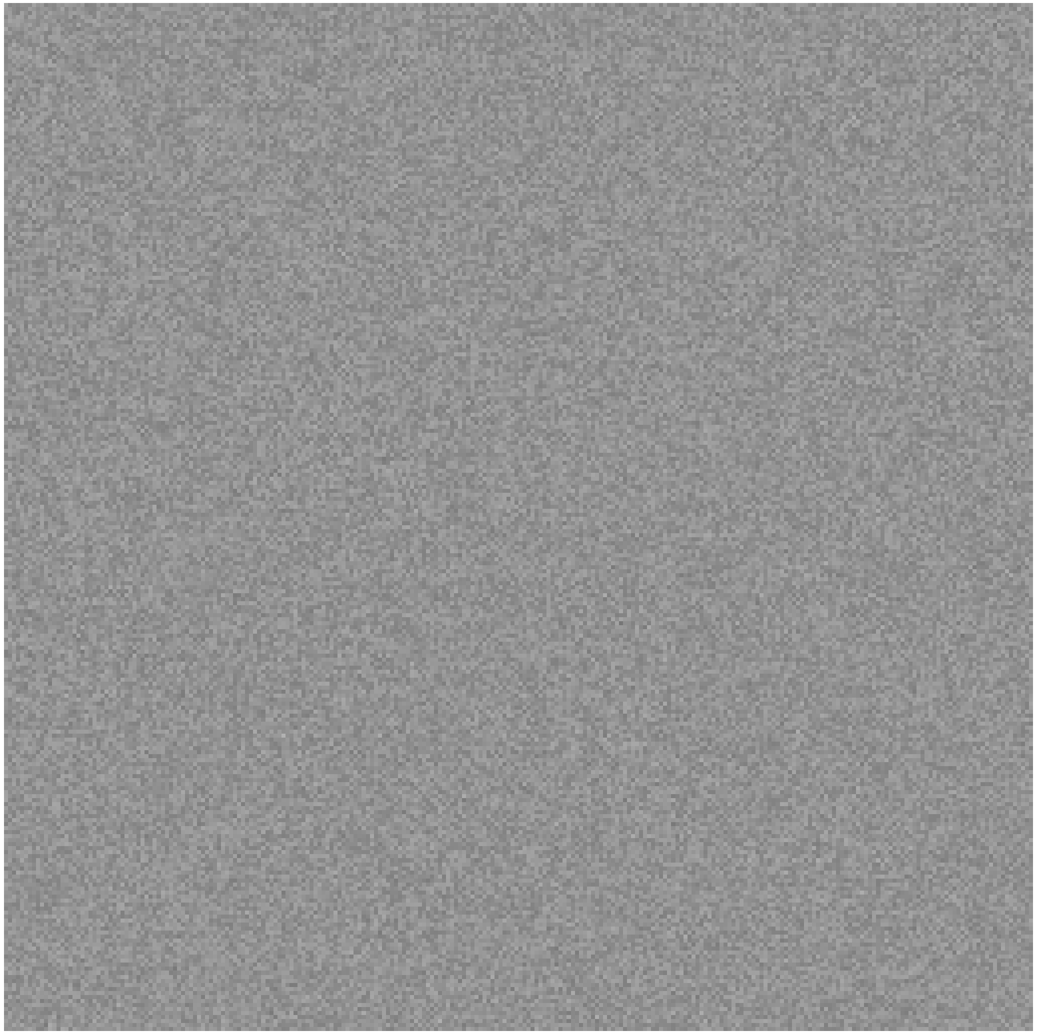
$a = 0$ $D_u = 1, D_v = 5$



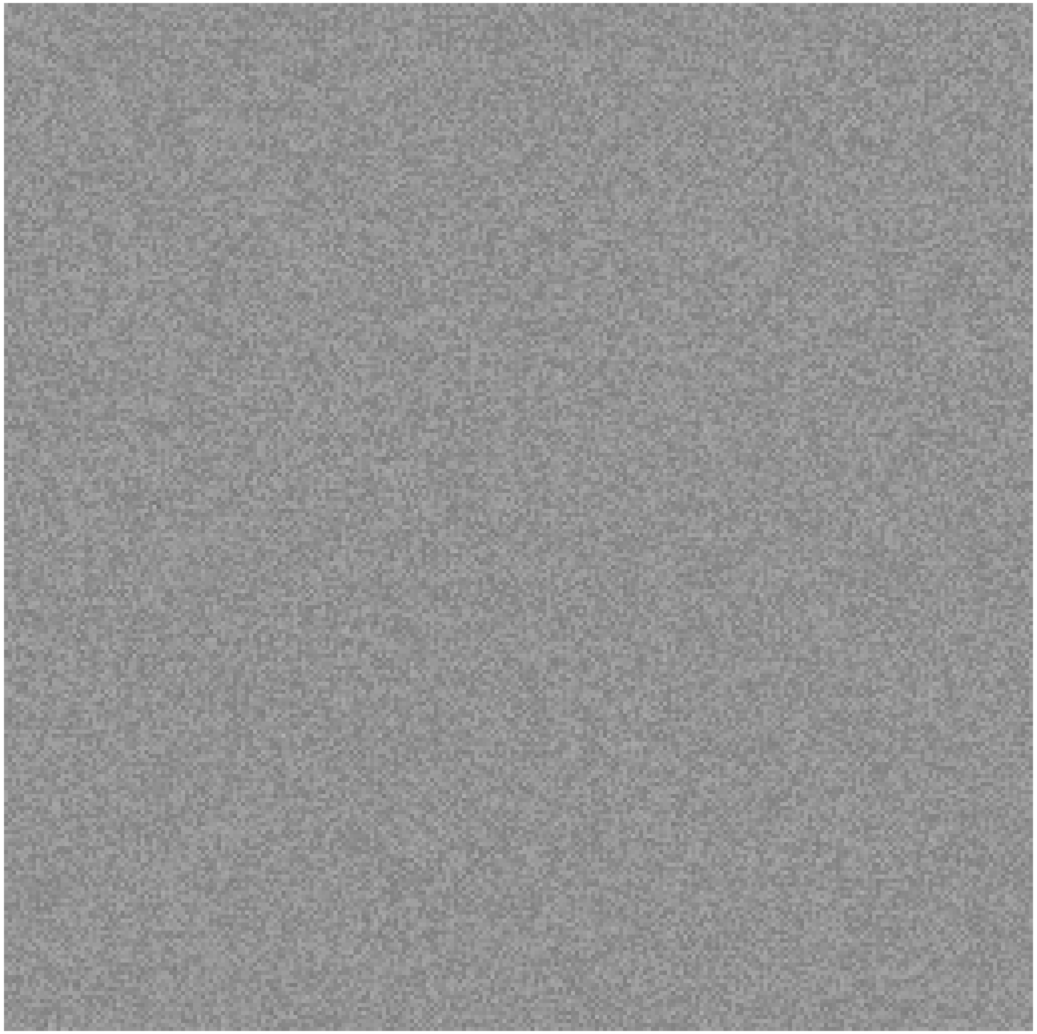
$a = 0$ $D_u = 1, D_v = 20$



$a = 0$ $D_u = 2, D_v = 20$



$a = 0.05$ $D_u = 1, D_v = 20$



Turing Pattern

論文:

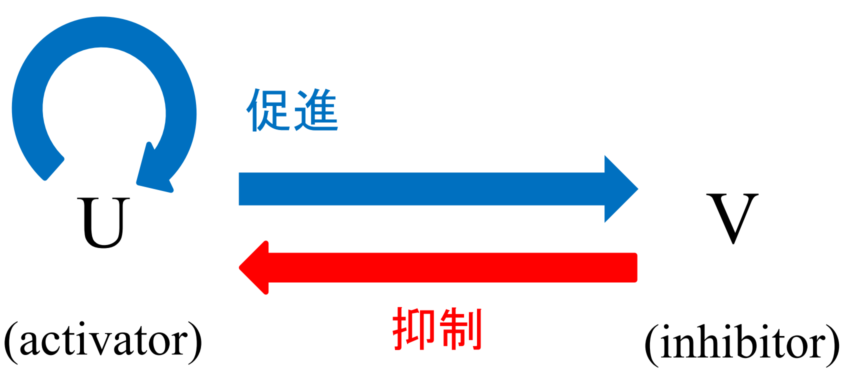
THE CHEMICAL BASIS OF MORPHOGENESIS

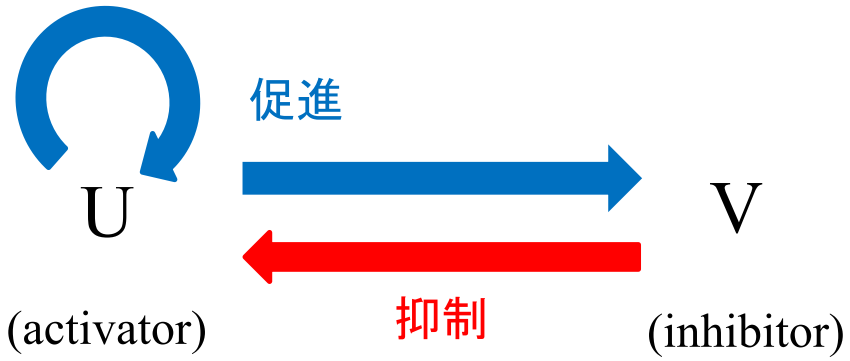
By A. M. TURING, F.R.S. *University of Manchester*

(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

A.M. Turing, Phil. Trans. R. Soc. Lond. B **237**, 37 (1952).

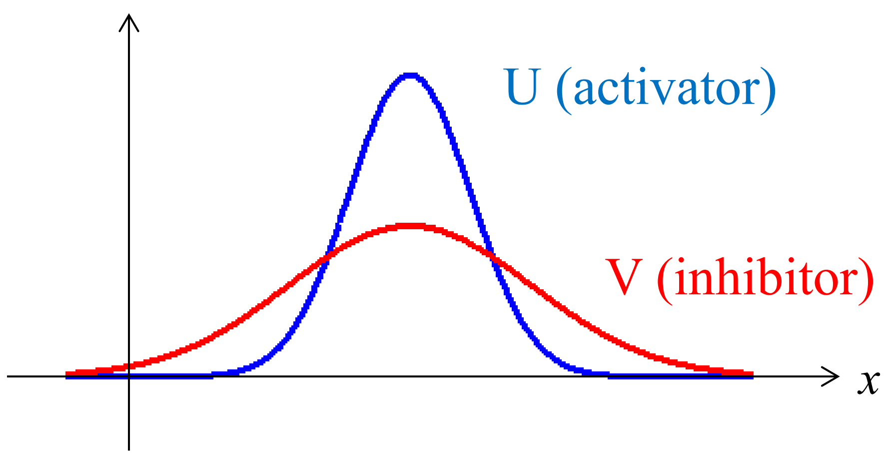
morphogen（形態形成因子）のダイナミクスを考える





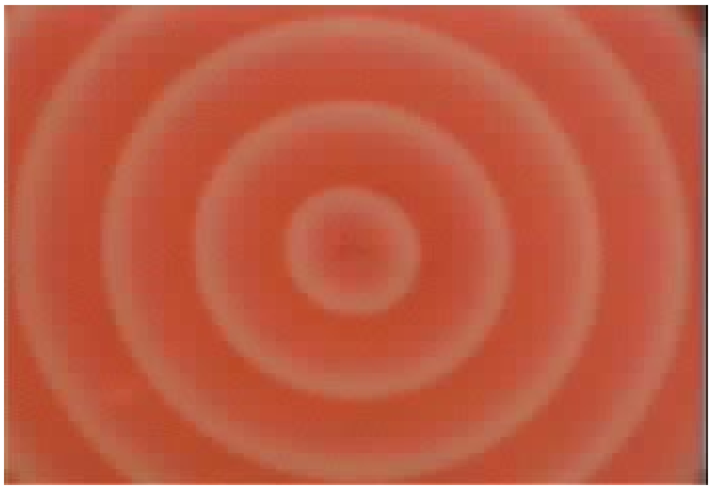
撹拌されていれば･･･:　一様な安定状態で落ち着く

Vの拡散がUより速い:　Uが多い領域ができ、その周りにVが多い領域が分布



Belousov-Zhabotinsky反応

a) Target Pattern



（4倍速）

3 mm

b) Spiral Pattern

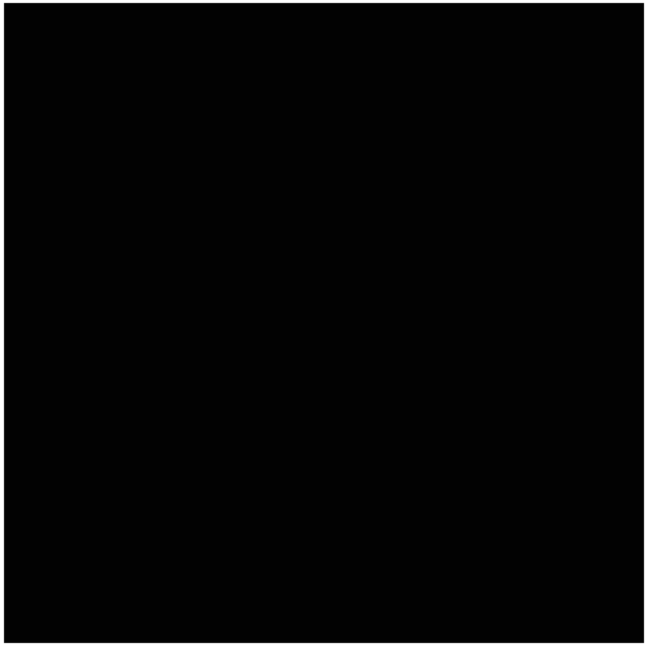
（4倍速）

3 mm

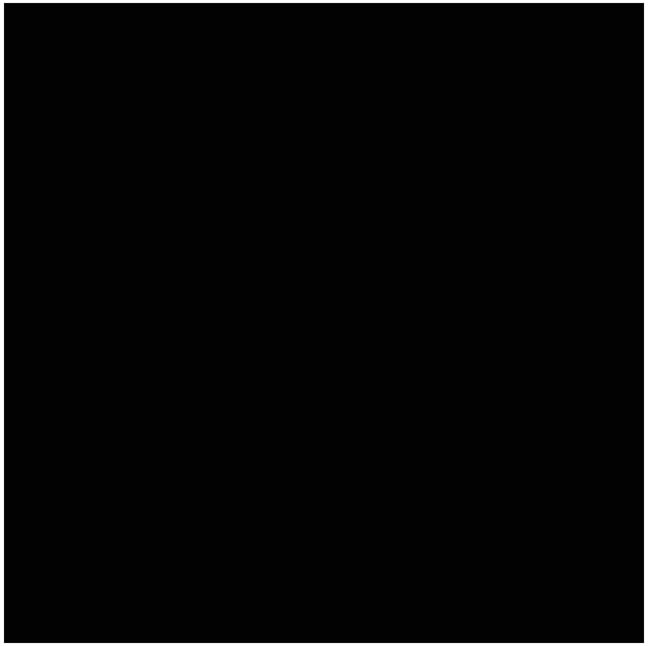
$$\frac{\partial U}{\partial t} = \frac{1}{\varepsilon}\left(U(1-U) - fV\frac{U-q}{U+q} \right) + D_U \nabla^2 U$$

$$\frac{\partial V}{\partial t} = U - V + D_V \nabla^2 V$$

Target Pattern



Spiral Pattern



（Keener-Tyson version Oregonatorを使用）