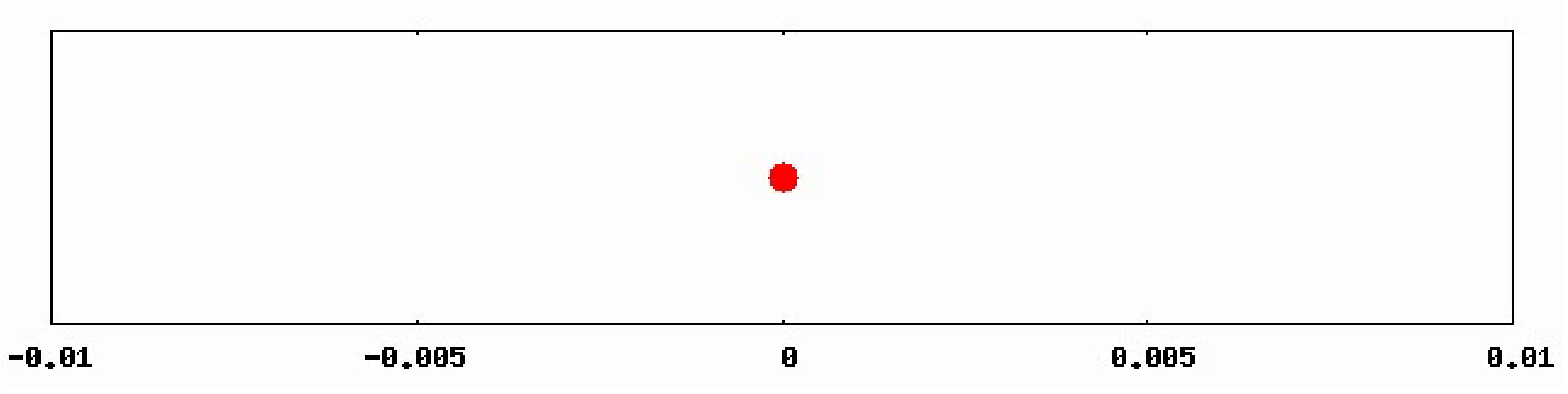
Langevin方程式

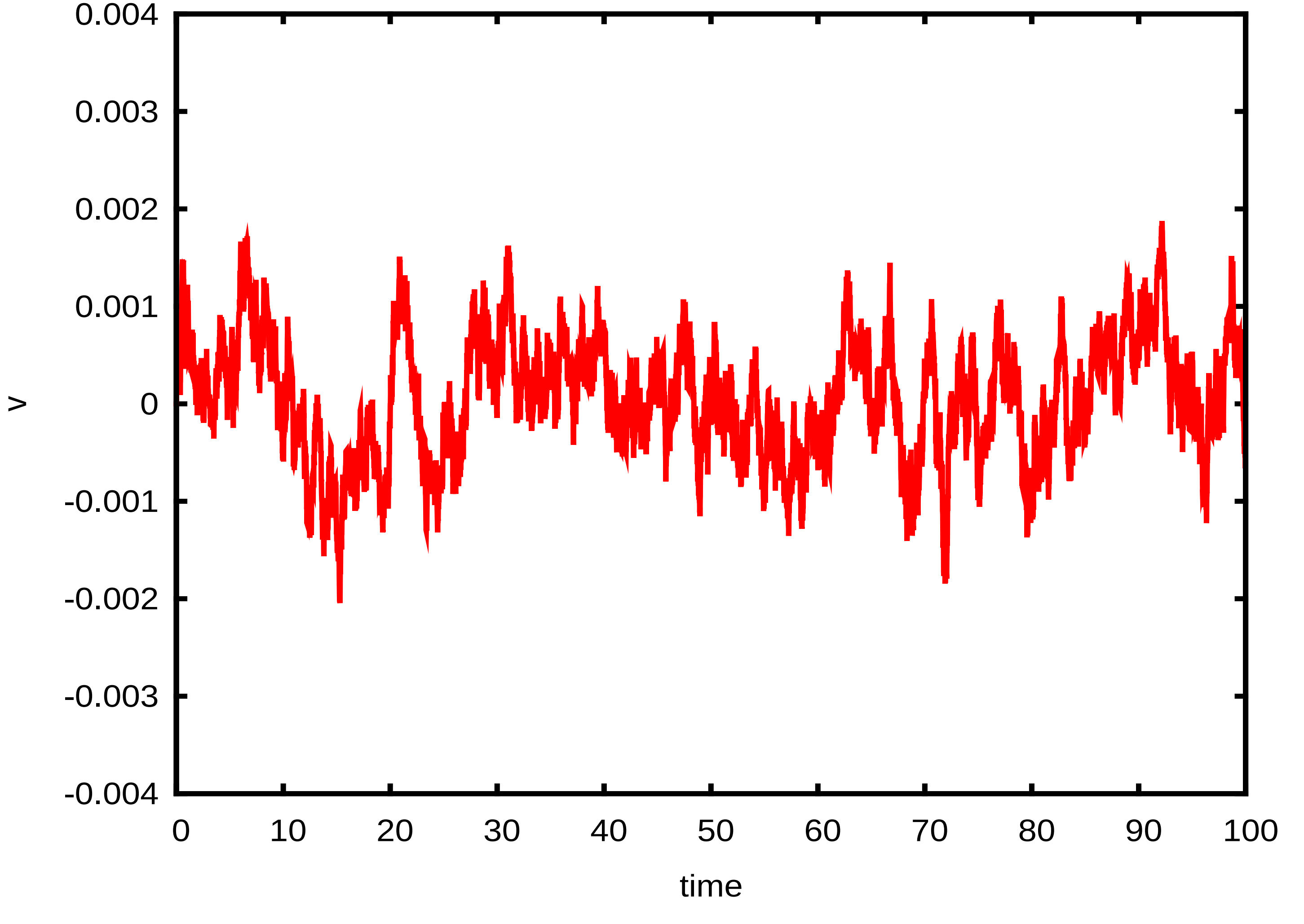
$$m\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = -k\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \boldsymbol{\xi}(t)$$

$$\left\langle \boldsymbol{\xi}(t) \right\rangle = 0$$

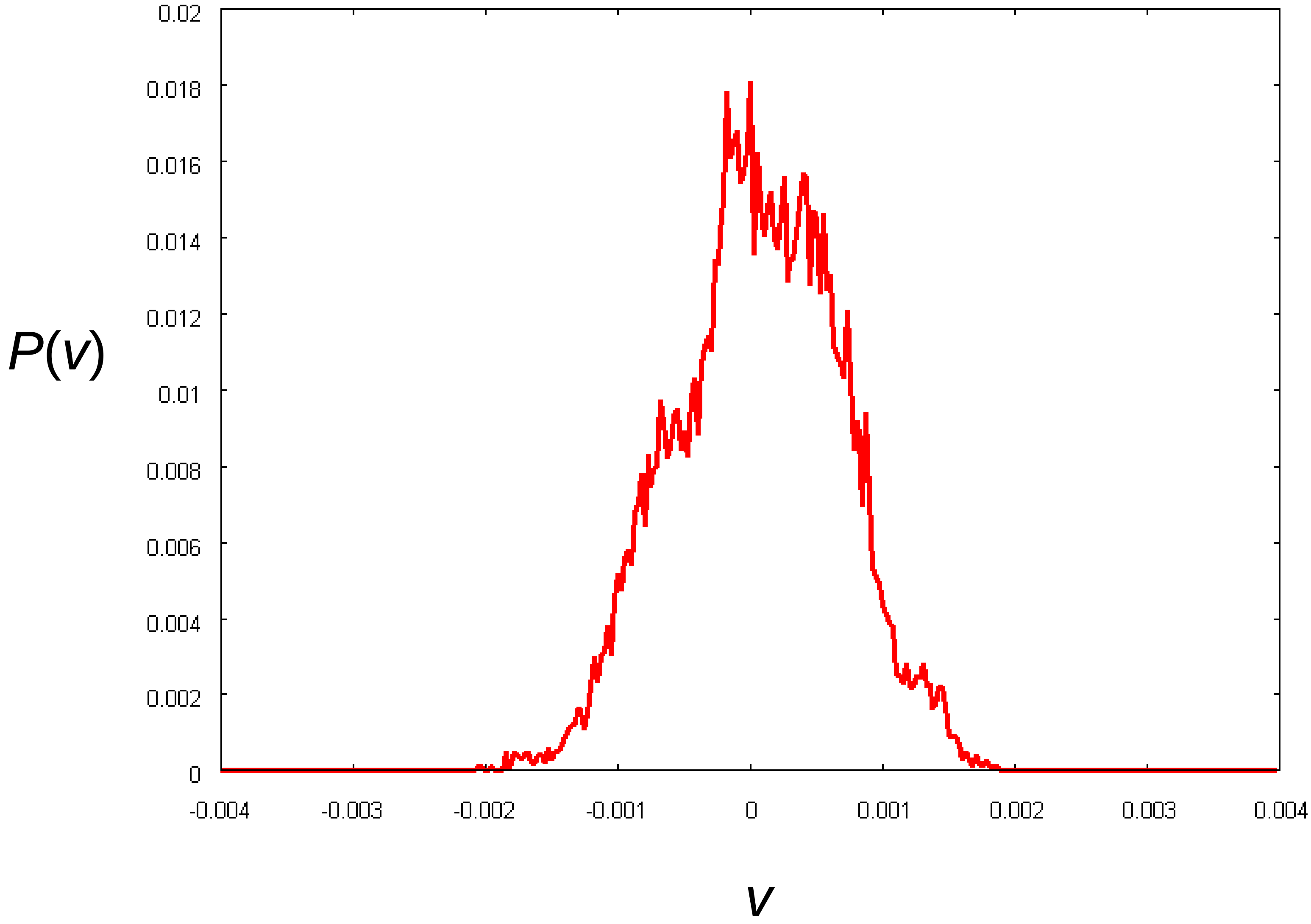
$$\left\langle \boldsymbol{\xi}(t)\cdot\boldsymbol{\xi}(s) \right\rangle = M\delta\left(t-s\right)$$

1次元系でのLangevin方程式による挙動




0.004
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-0.004
v
0
10
20
30
40
50
60
70
80
90
100
time

速度の分布



P(v)
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
v

外力があるときのLangevin方程式

$$m\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = -k\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \mathbf{F}(\mathbf{r}) + \boldsymbol{\xi}(t)$$

$$\left\langle \boldsymbol{\xi}(t) \right\rangle = 0$$

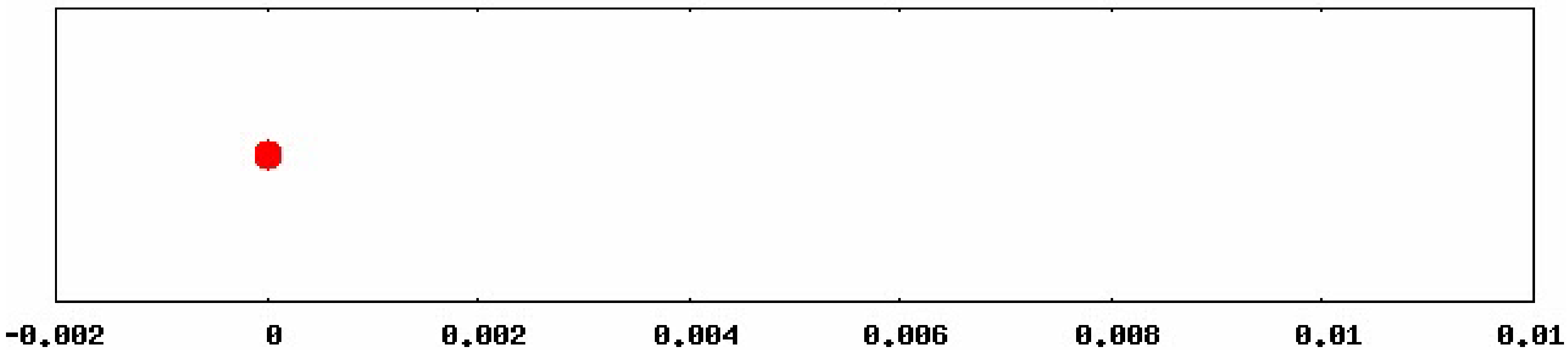
$$\left\langle \boldsymbol{\xi}(t)\cdot\boldsymbol{\xi}(s) \right\rangle = M\delta\left(t-s\right)$$

1次元で一定の外力の場合

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -k\frac{\mathrm{d}x}{\mathrm{d}t} + F + \xi(t)$$

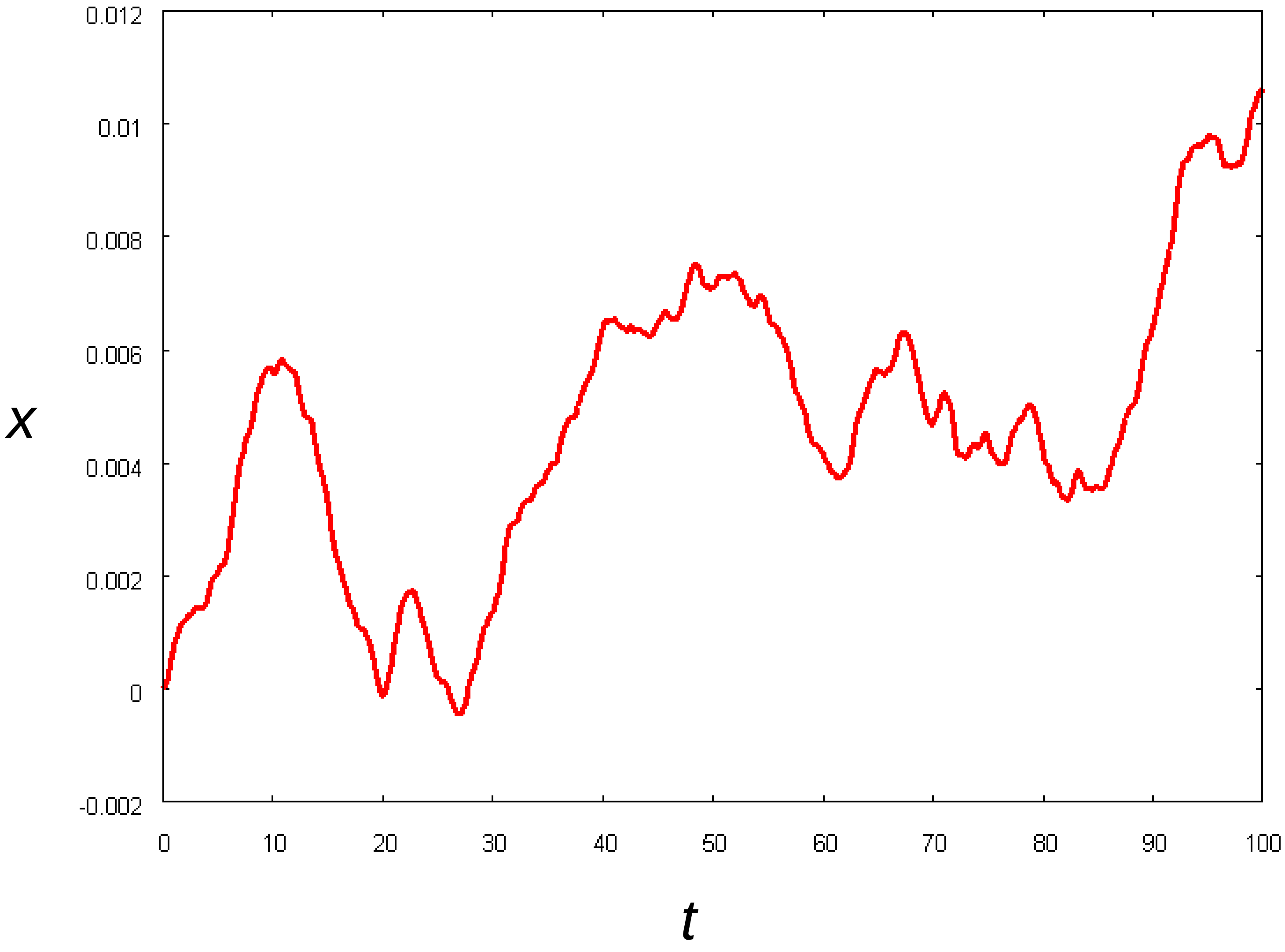
$$\left\langle \xi(t) \right\rangle = 0$$

$$\left\langle \xi(t)\cdot\xi(s) \right\rangle = M\delta\left(t-s\right)$$

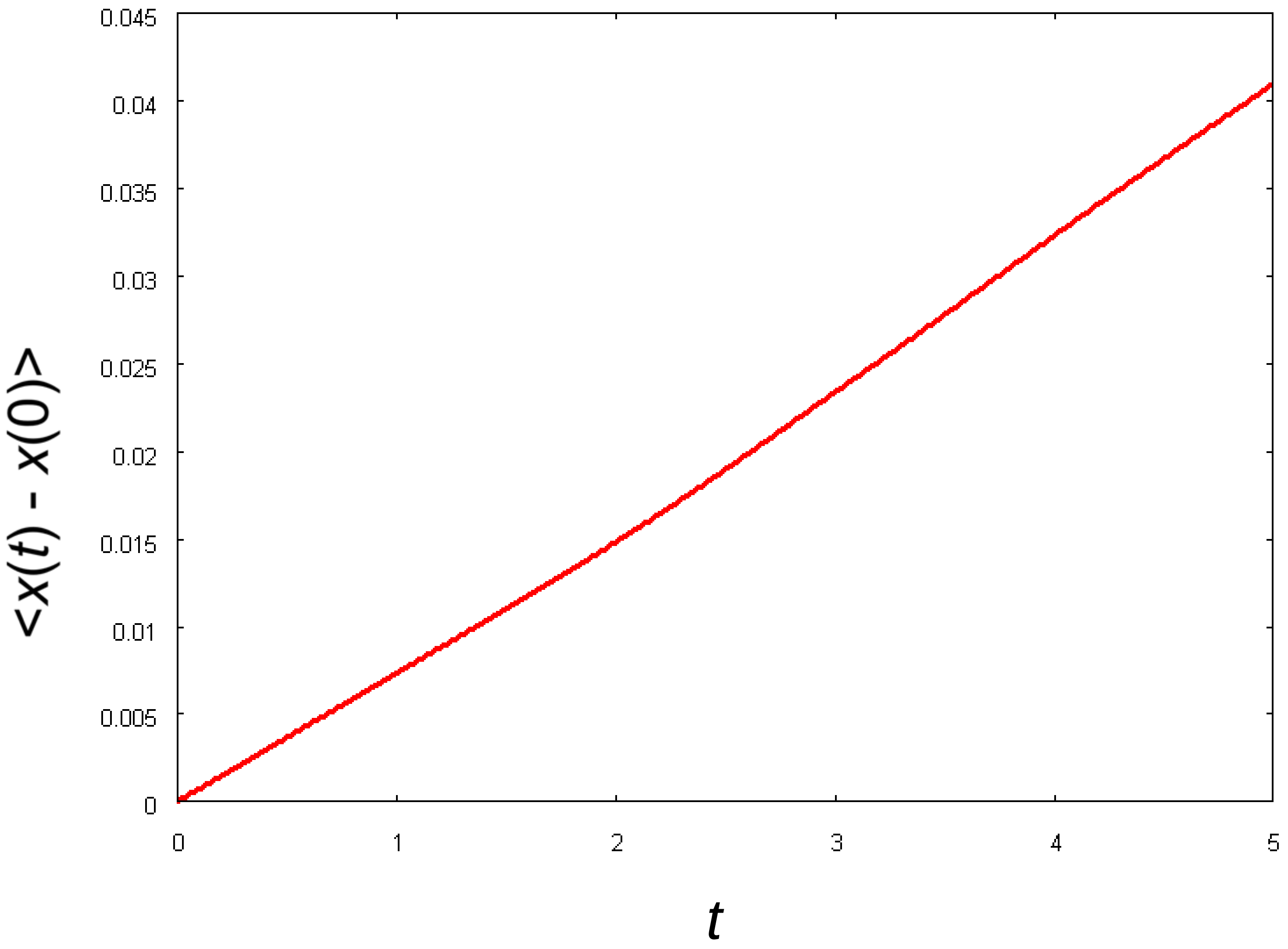


-0.002
0
0.002
0.004
0.006
0.008
0.01
0.01

$x(t)$ vs. t


0.012
0.01
0.008
0.006
0.004
0.002
0
-0.002
x
0
10
20
30
40
50
60
70
80
90
100
t

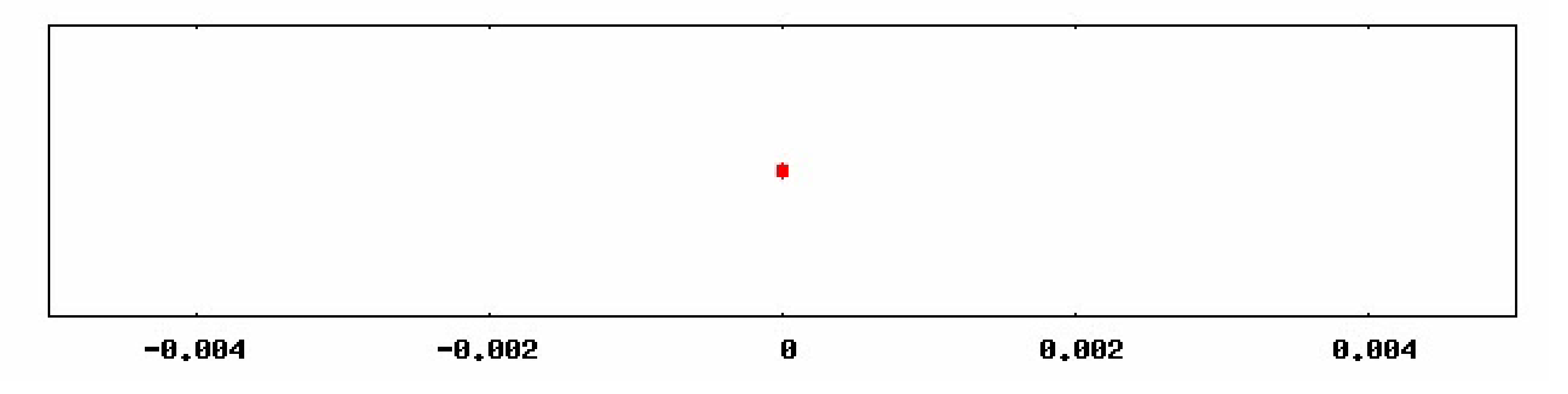
$\langle x(t) - x(0) \rangle$ vs. t



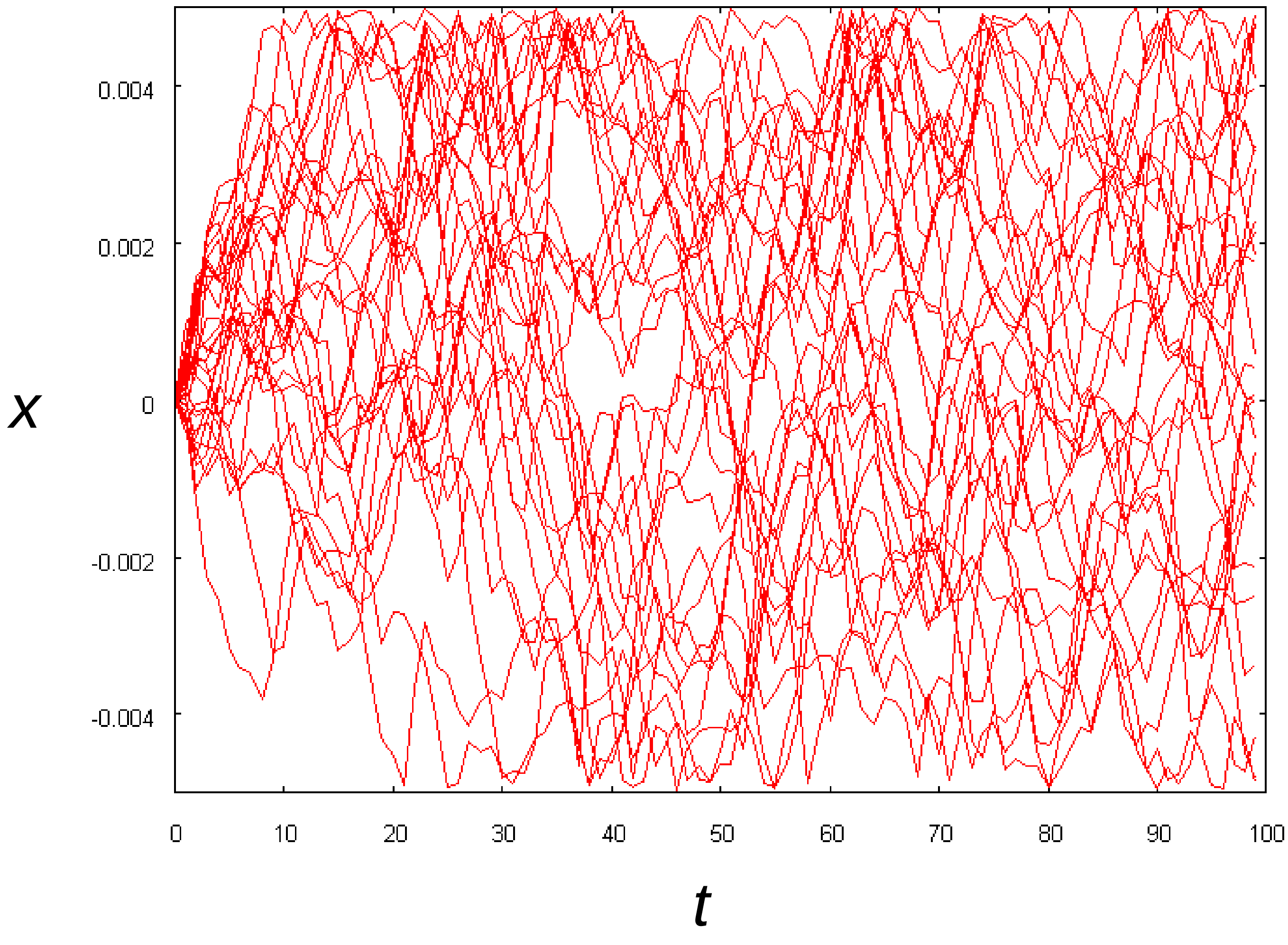
$$\langle v(t) \rangle = F \,/\, \gamma$$

1次元で一定の外力の場合

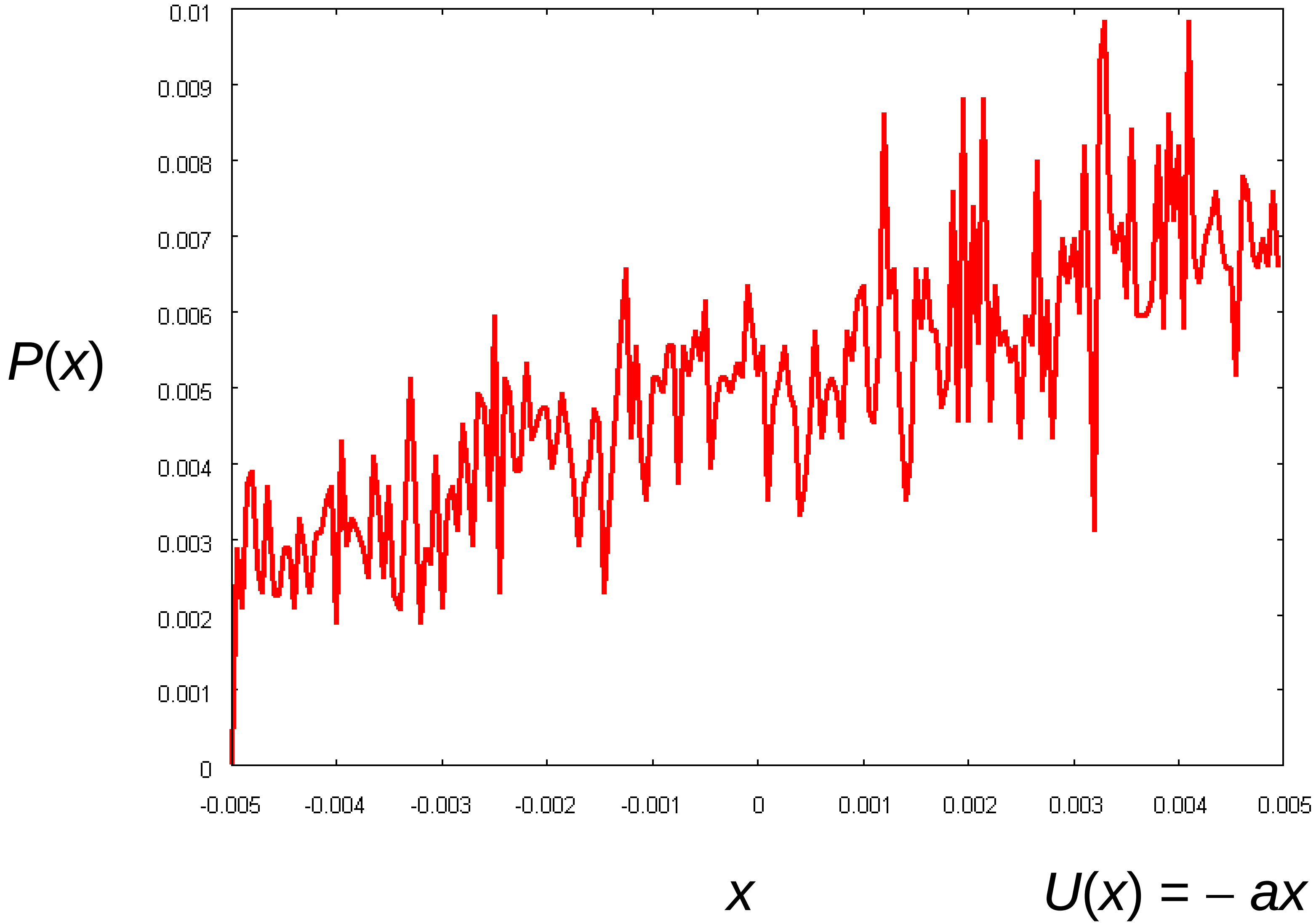
（多くの粒子を入れて、領域を区切ると･･･）



x


0.004
0.002
0
-0.002
-0.004
x
0
10
20
30
40
50
60
70
80
90
100
t

分 布


0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
P(x)
-0.005
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
0.005
x

$U(x) = -ax$

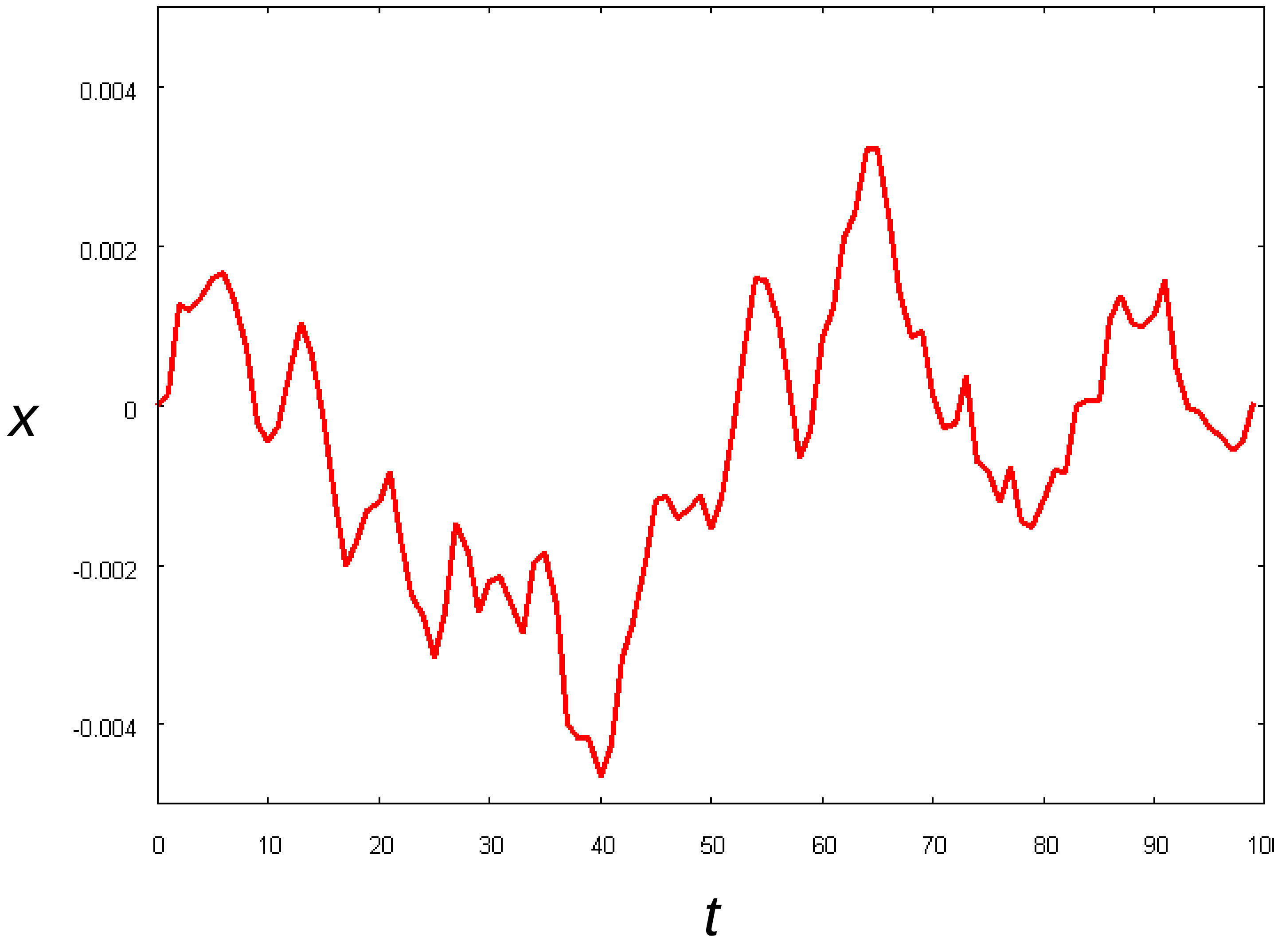
1次元で調和ポテンシャル中の場合

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -k\frac{\mathrm{d}x}{\mathrm{d}t} - ax + \xi(t)$$

$$\Phi(x) = \frac{a}{2}x^2$$

$$\left\langle \xi(t) \right\rangle = 0$$

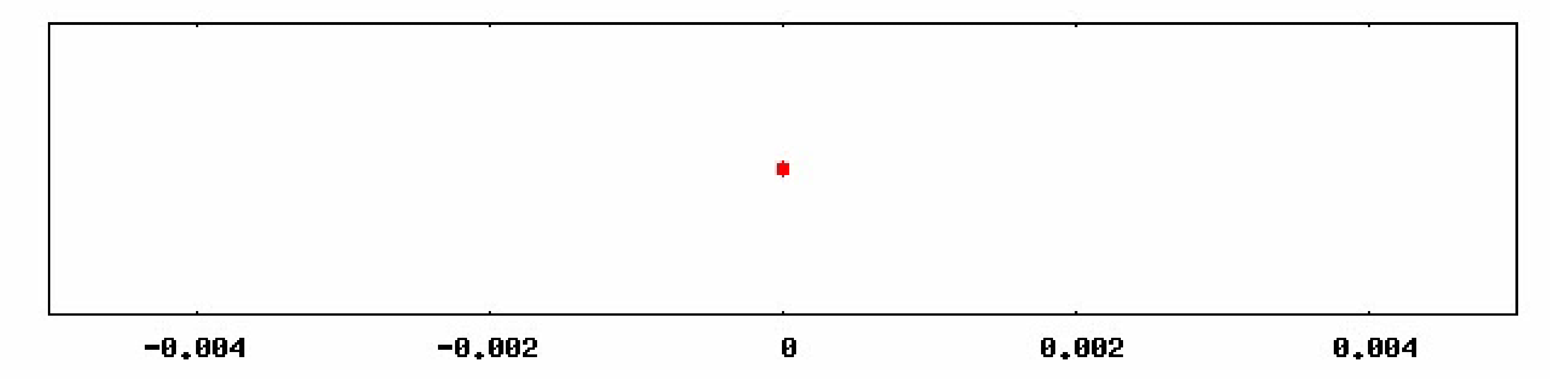
$$\left\langle \xi(t)\cdot\xi(s) \right\rangle = M\delta\left(t-s\right)$$



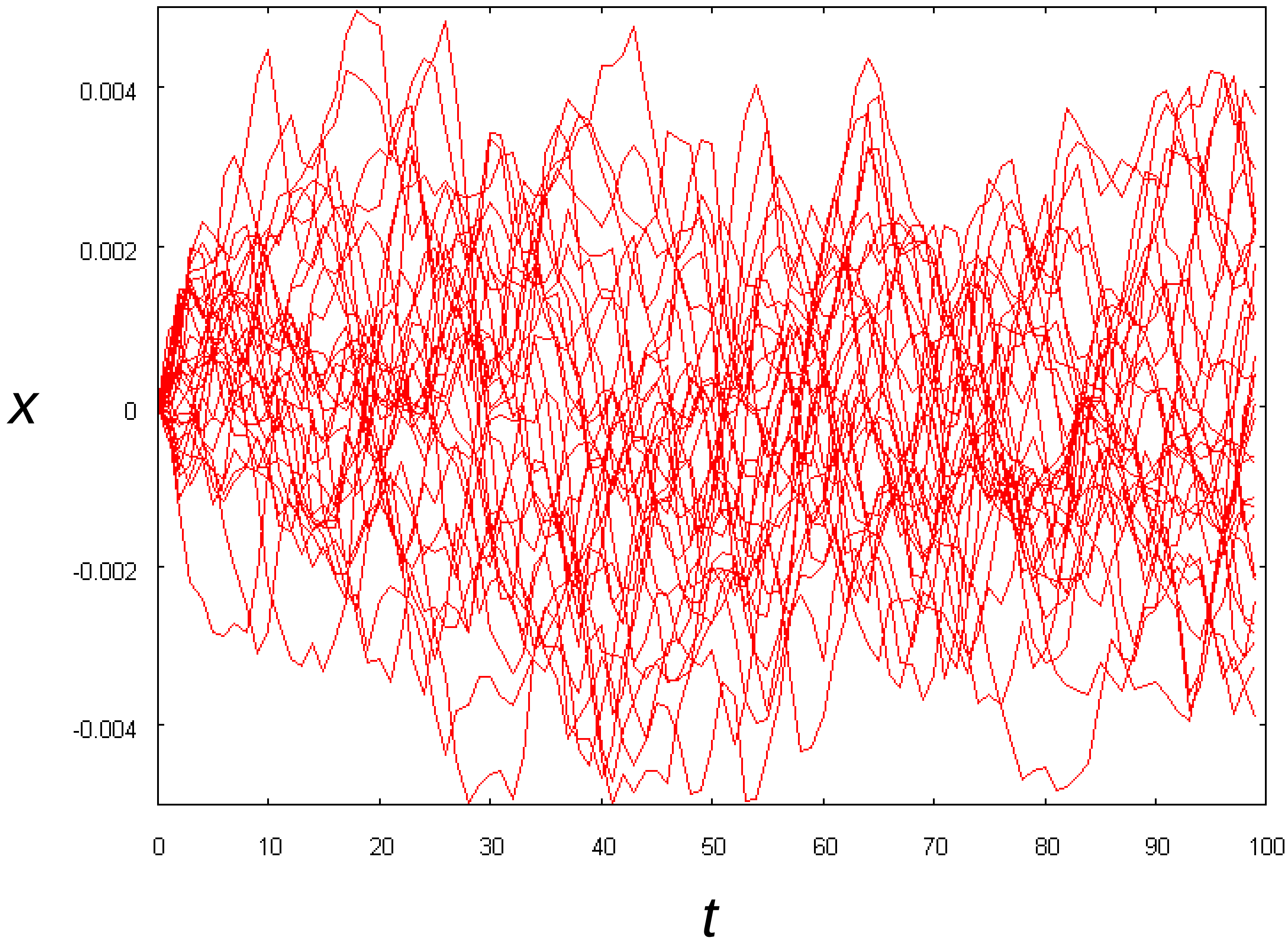
0.004
0.002
0
-0.002
-0.004
x
0
10
20
30
40
50
60
70
80
90
100
t

1次元で調和ポテンシャル中の場合

（多くの粒子を入れると･･･）

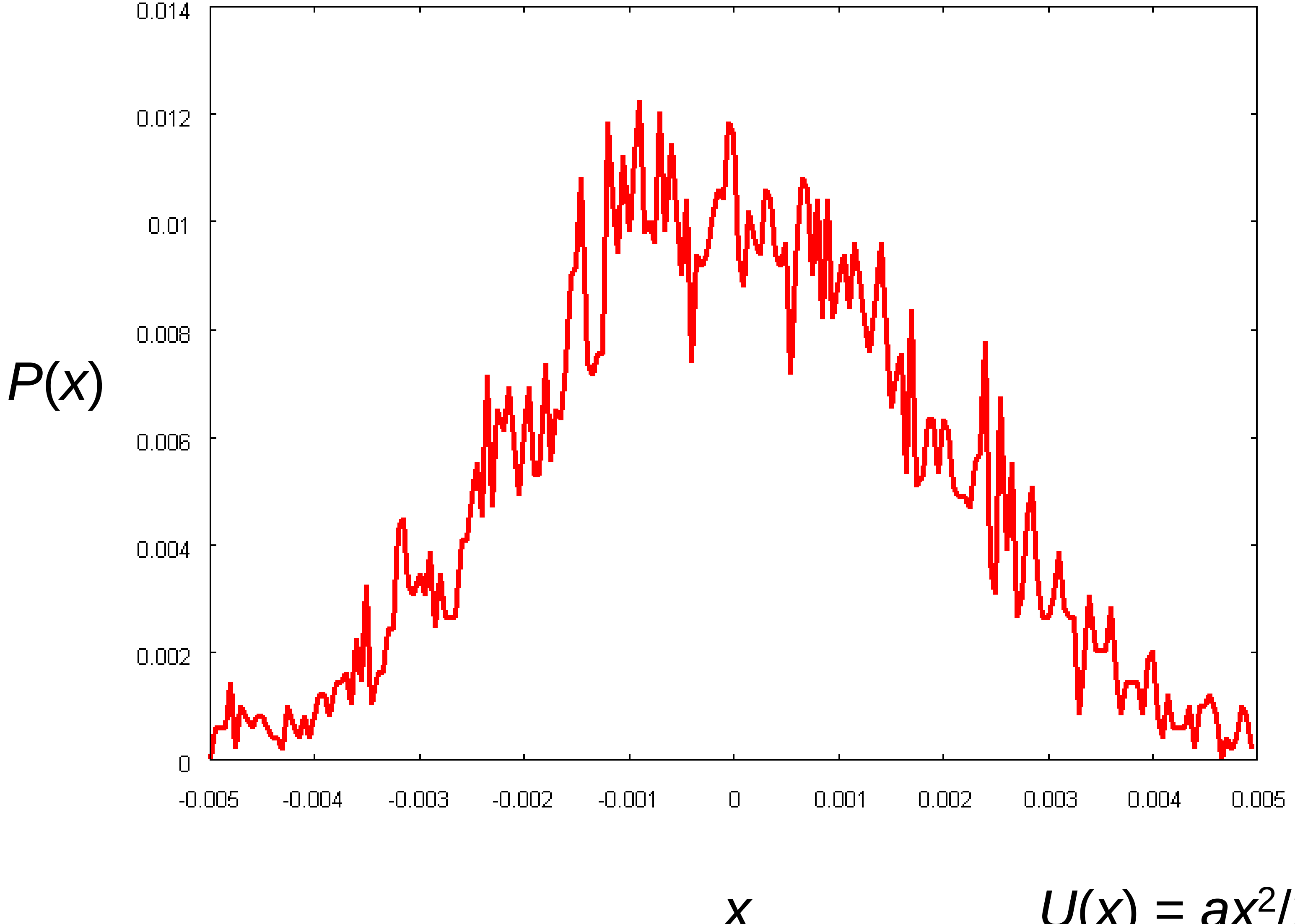


x



x
0.004
0.002
0
-0.002
-0.004
0
10
20
30
40
50
60
70
80
90
100
t

分 布


0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
-0.005
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
0.005
P(x)
x

$U(x) = ax^2/2$