分岐理論

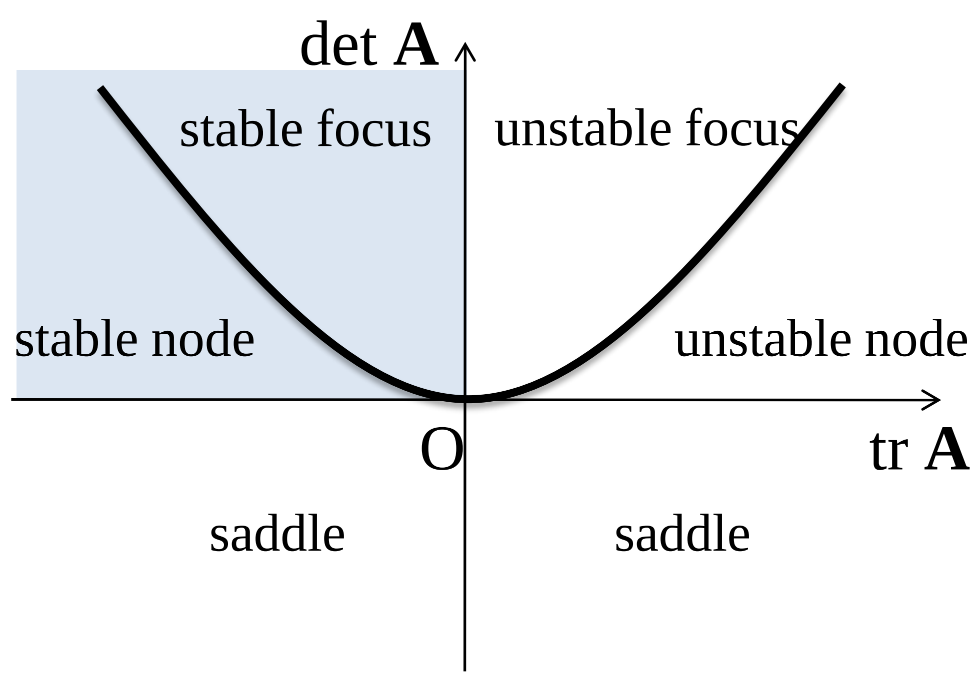
おもな分岐の種類

- サドル・ノード分岐
- ピッチフォーク分岐
- 安定性交替分岐（トランスクリティカル分岐）
- ホップ分岐

分岐とは?　　2変数の力学系で考える

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix}$$

$\mathbf{A}$: 関数行列



ただし、ホップ分岐以外では、x方向とy方向は独立しているので、
$\mathrm{d}y/\mathrm{d}t = a\,y$　として考えることにする。

- サドル・ノード分岐

$$u = \pm\sqrt{\alpha}, v = 0 \ (\alpha \geq 0)$$ まわりで

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \alpha - u^2$$

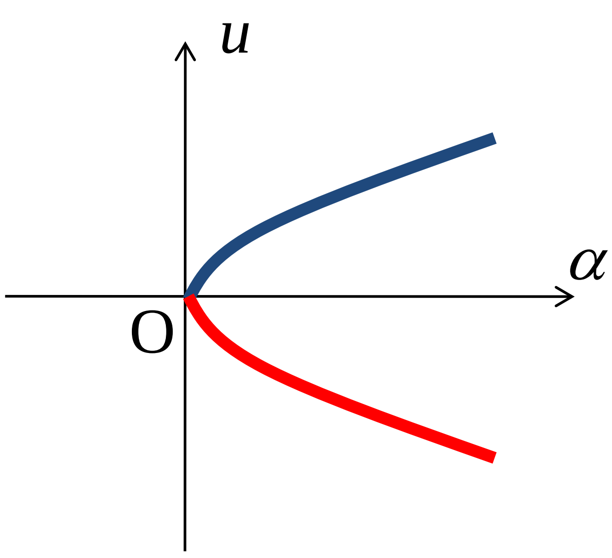
$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$

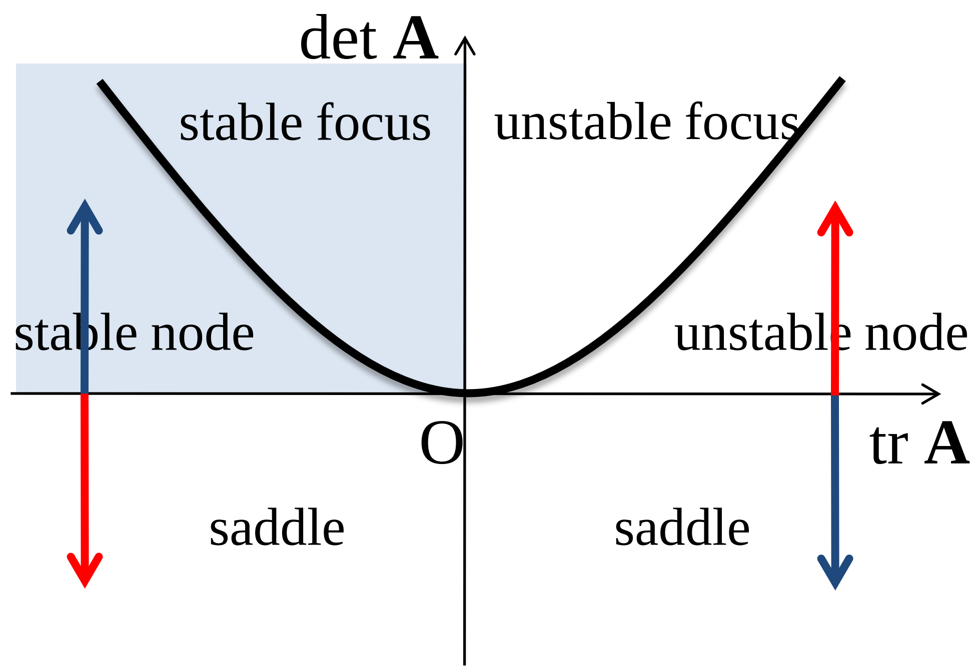
固定点：

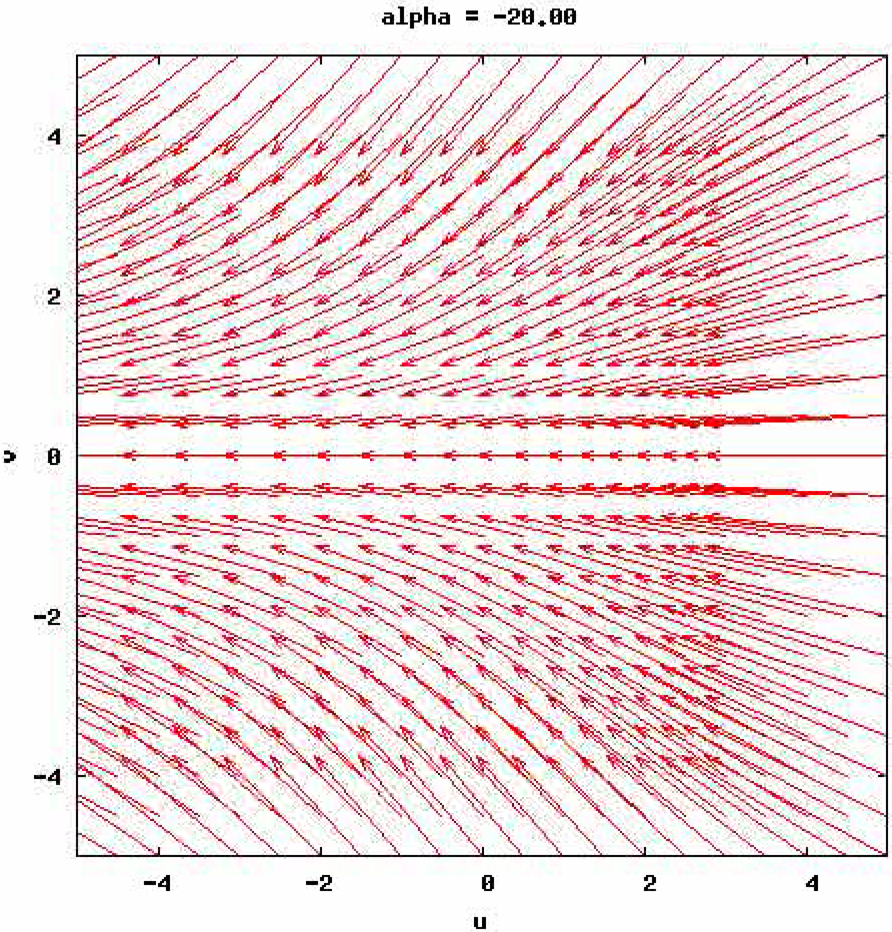
$$u = \pm\sqrt{\alpha}$$

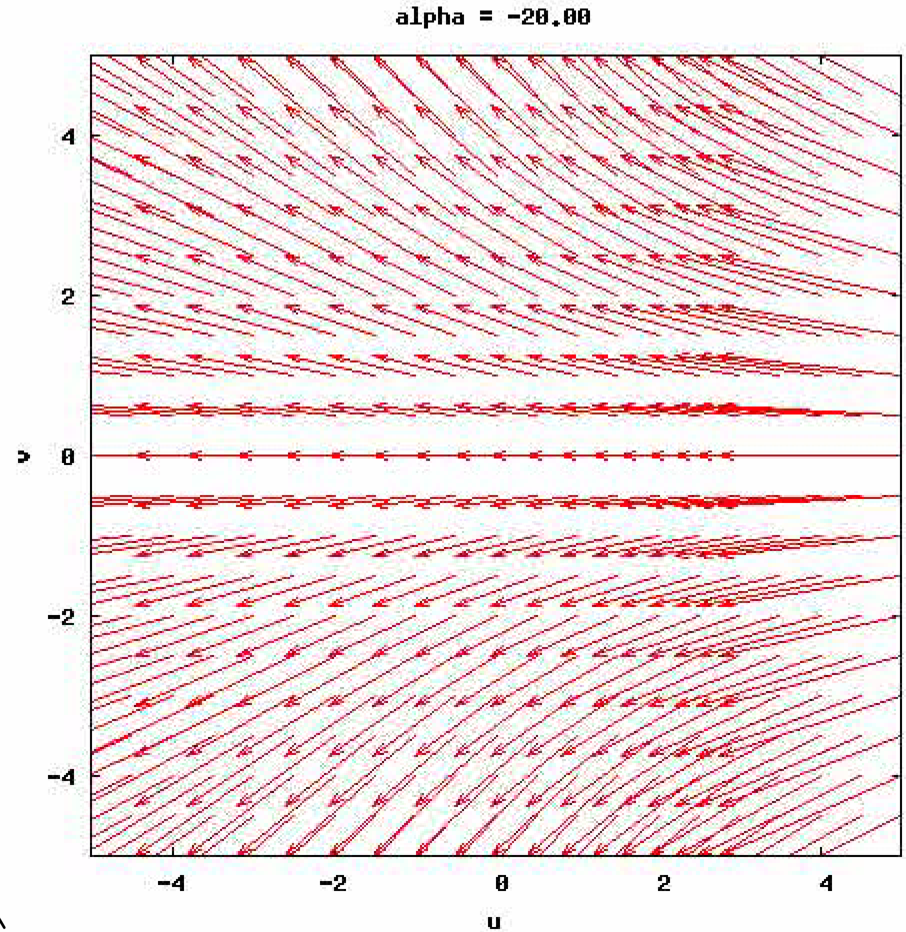
$$v = 0$$

$$\mathbf{A} = \begin{pmatrix} \mp 2\sqrt{\alpha} & 0 \\ 0 & k \end{pmatrix}$$





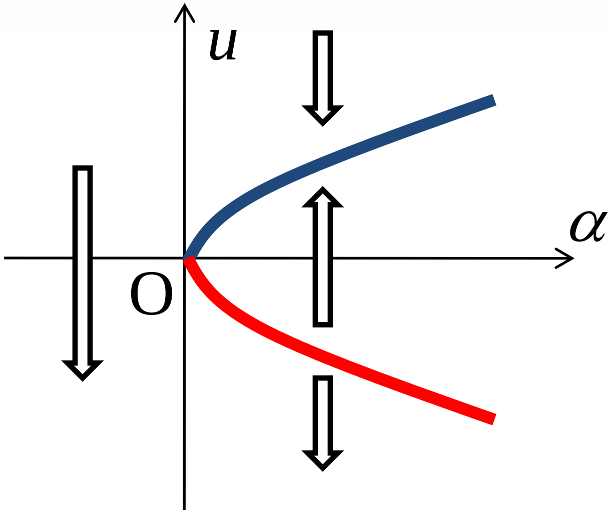




$$\frac{\mathrm{d}u}{\mathrm{d}t} = \alpha - u^2$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$

$k = -5$



$k = 5$

- ピッチフォーク分岐

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \alpha u - u^3$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$

固定点：

$$u = 0, \pm\sqrt{\alpha}$$

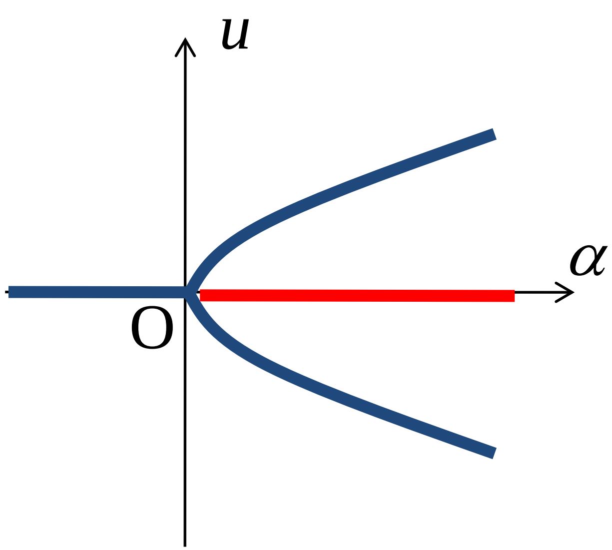
$$v = 0$$

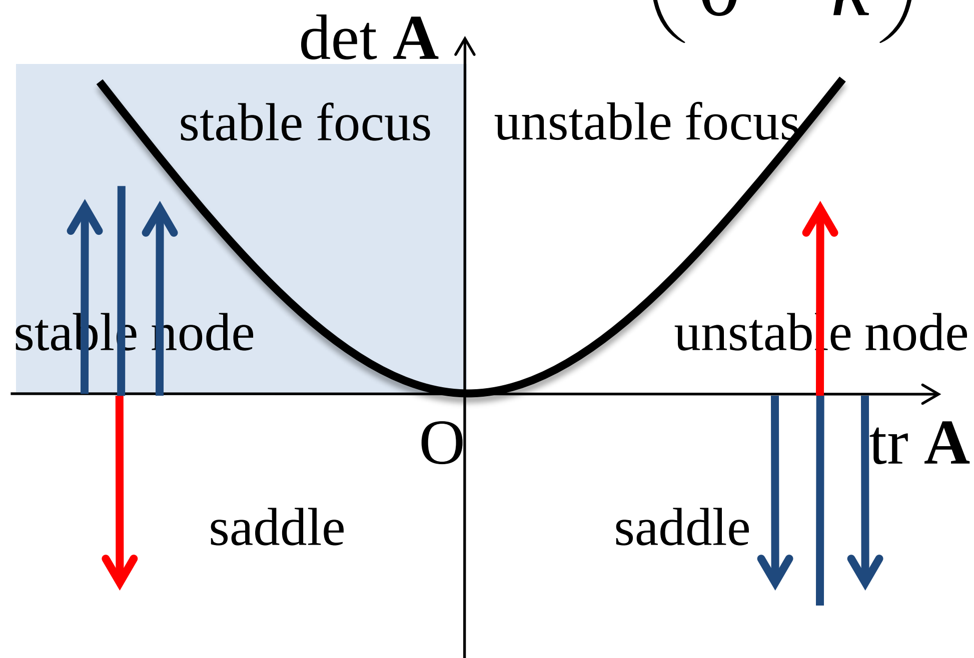
$u = \pm\sqrt{\alpha}, v = 0 \ (\alpha \geq 0)$ まわりで

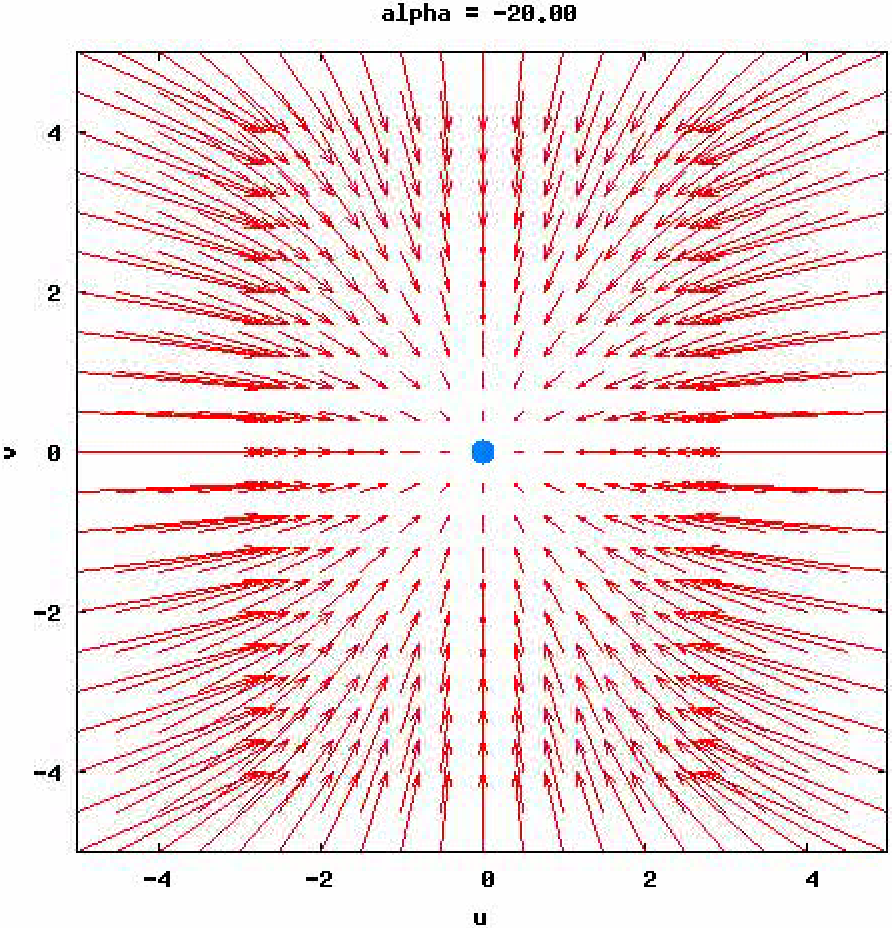
$$\mathbf{A} = \begin{pmatrix} -2\alpha & 0 \\ 0 & k \end{pmatrix}$$

$u = 0, v = 0$ まわりで

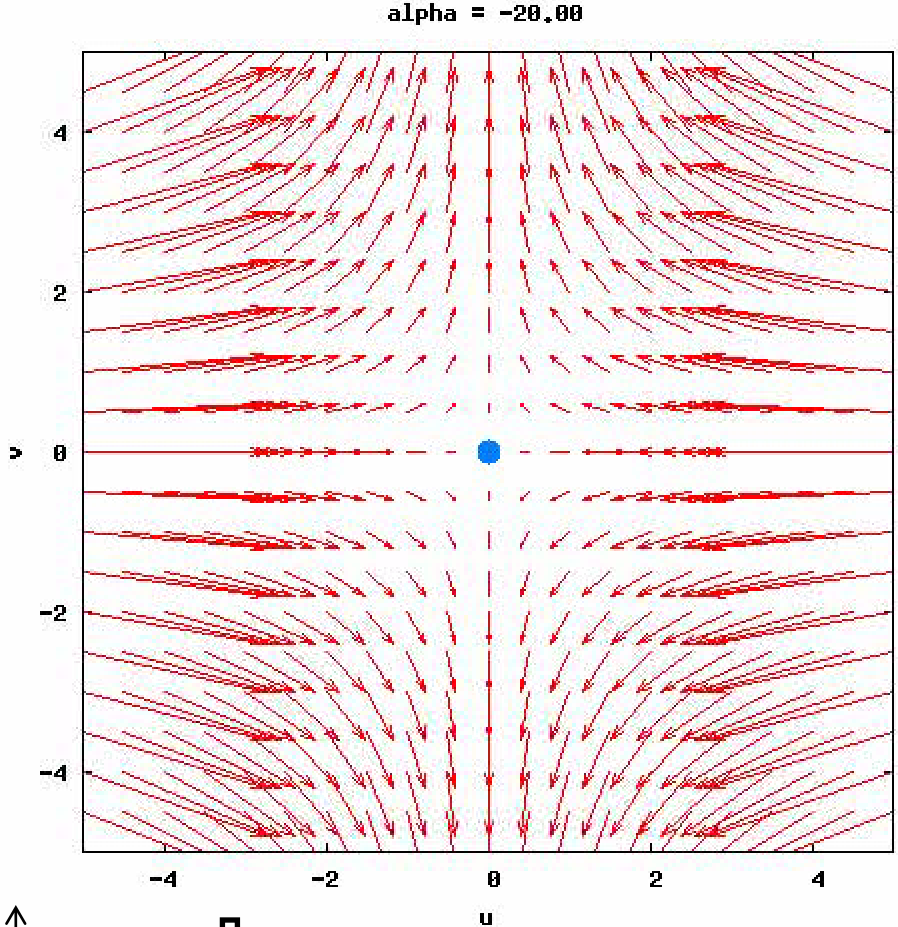
$$\mathbf{A} = \begin{pmatrix} \alpha & 0 \\ 0 & k \end{pmatrix}$$







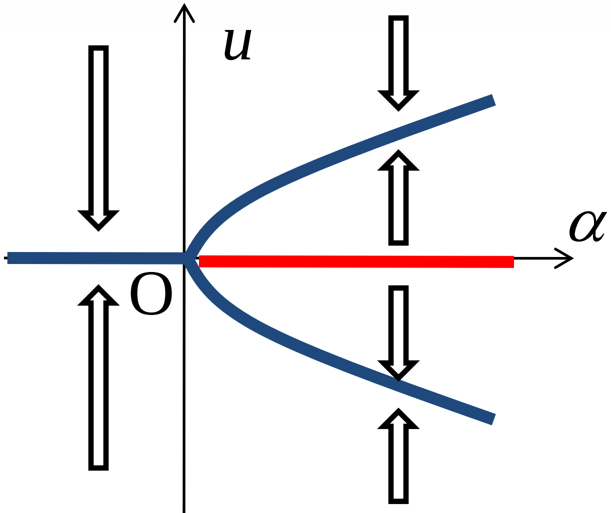
$k = -20$



$k = 20$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \alpha u - u^3$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$



- ピッチフォーク分岐（別タイプ）

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\alpha u + u^3$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$

固定点：

$$u = 0, \pm\sqrt{\alpha}$$

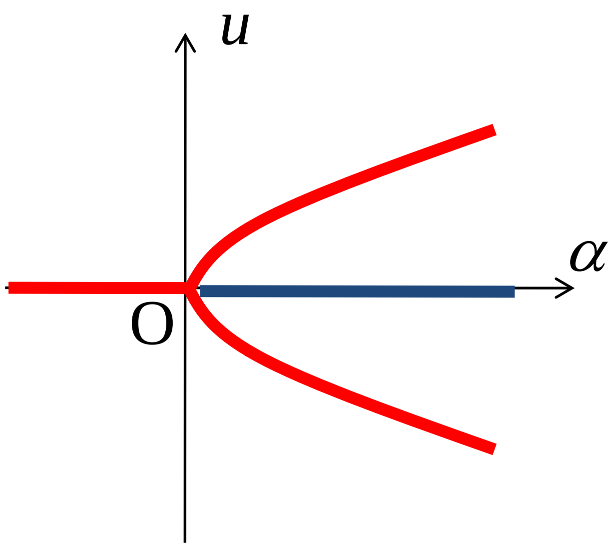
$$v = 0$$

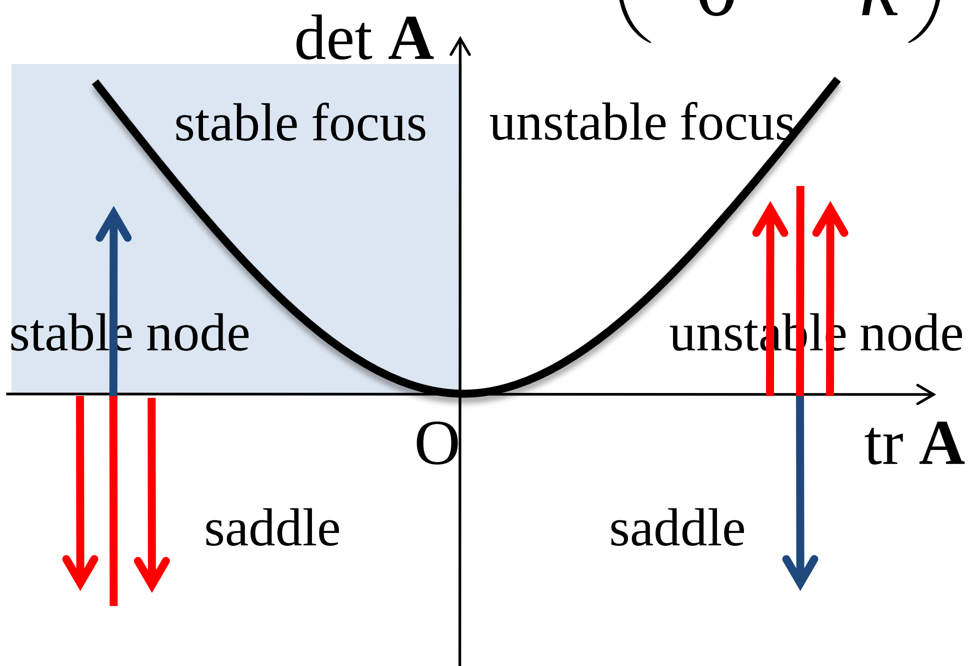
$u = \pm\sqrt{\alpha}, v = 0 \ (\alpha \geq 0)$ まわりで

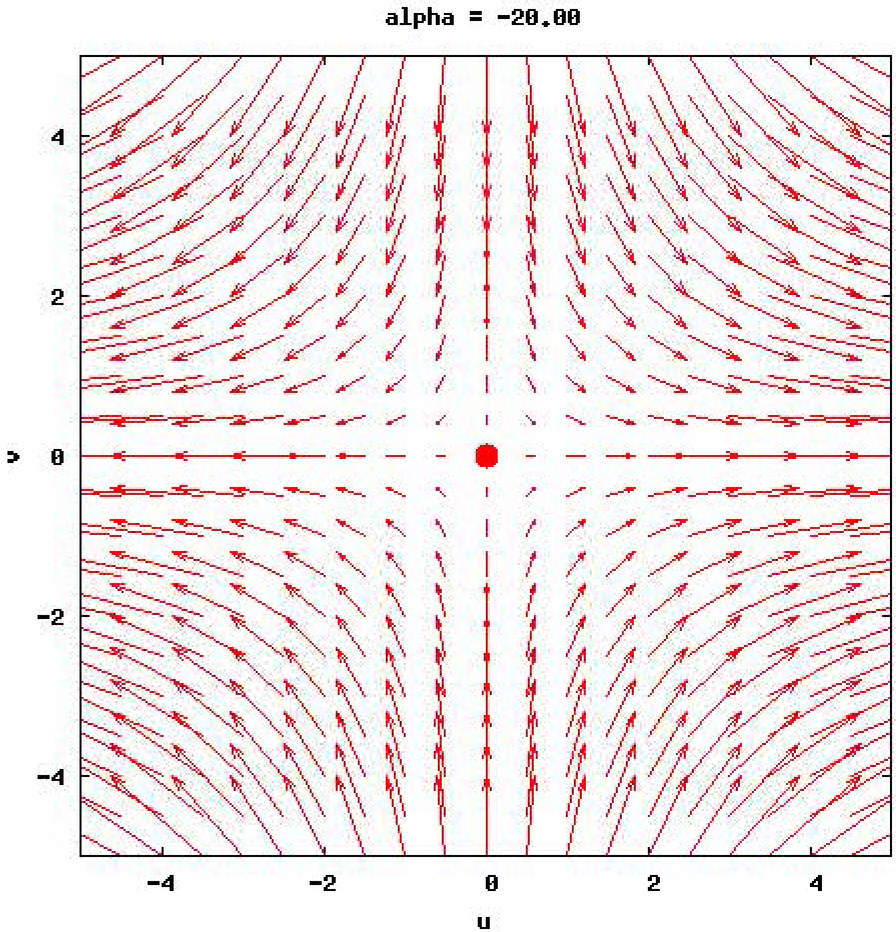
$$\mathbf{A} = \begin{pmatrix} 2\alpha & 0 \\ 0 & k \end{pmatrix}$$

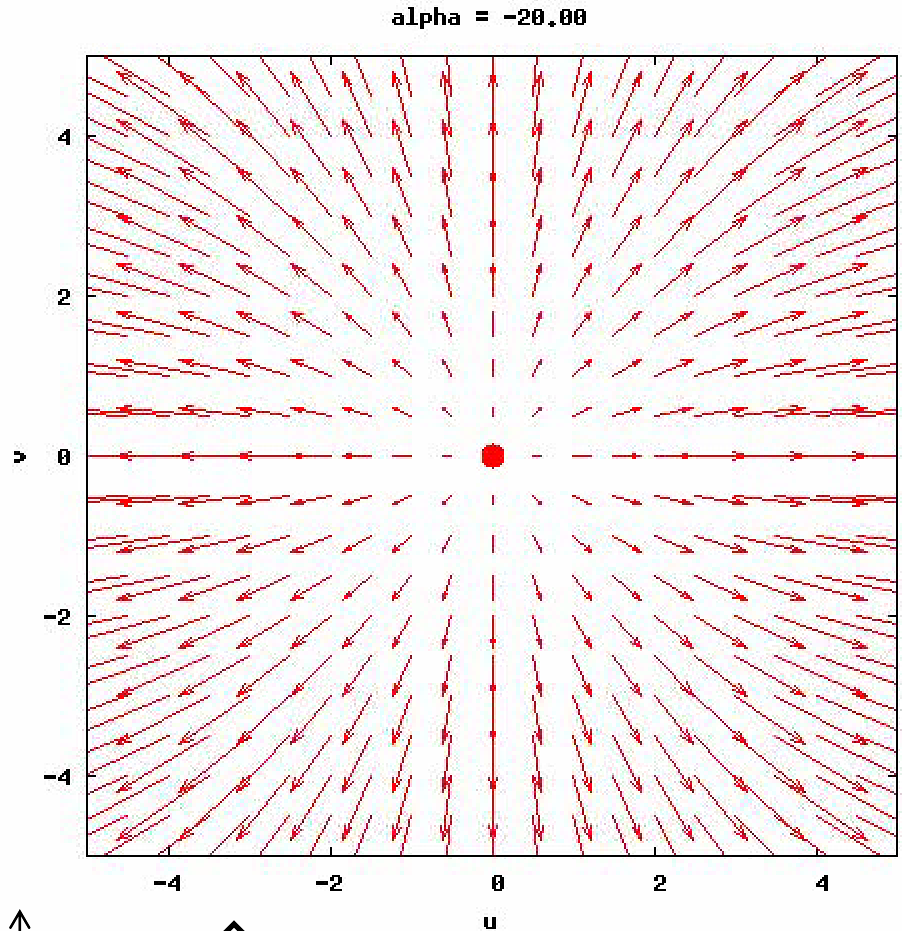
$u = 0, v = 0$ まわりで

$$\mathbf{A} = \begin{pmatrix} -\alpha & 0 \\ 0 & k \end{pmatrix}$$





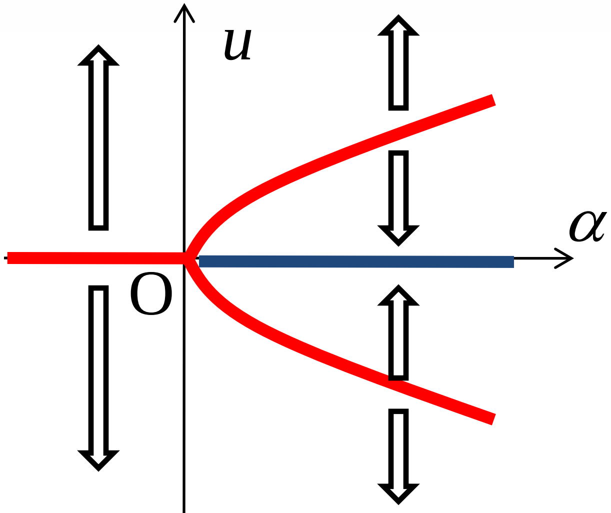




$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\alpha u + u^3$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$

$k = -20$



$k = 20$

- 安定性交替分岐

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \alpha u - u^2$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$

固定点：

$$u = 0,\ \lambda$$

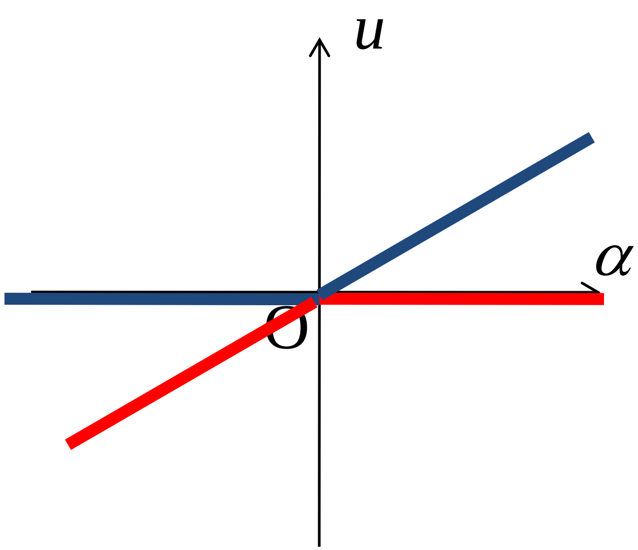
$$v = 0$$

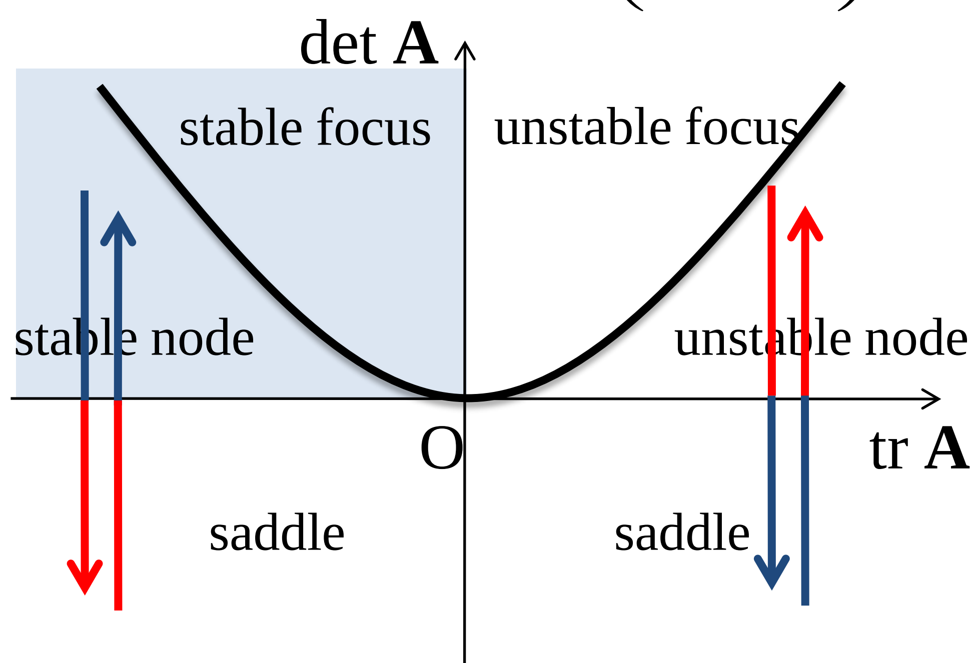
$u = \alpha, v = 0$ まわりで

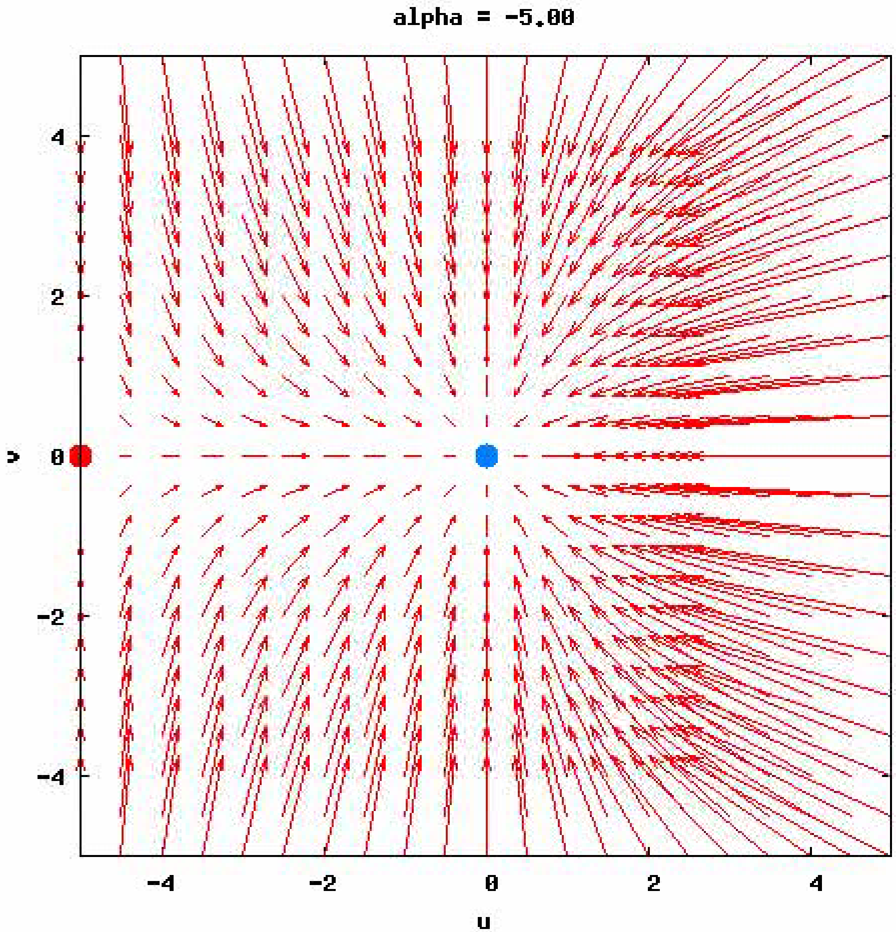
$$\mathbf{A} = \begin{pmatrix} -\alpha & 0 \\ 0 & k \end{pmatrix}$$

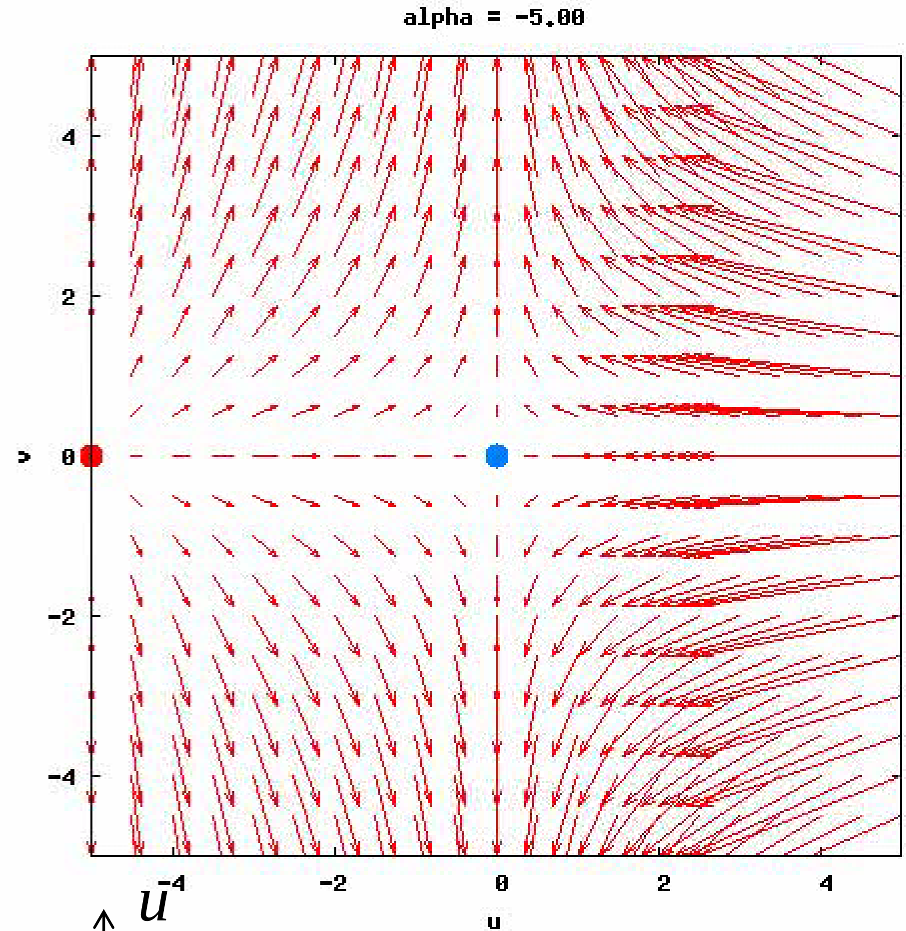
$u = 0, v = 0$ まわりで

$$\mathbf{A} = \begin{pmatrix} \alpha & 0 \\ 0 & k \end{pmatrix}$$





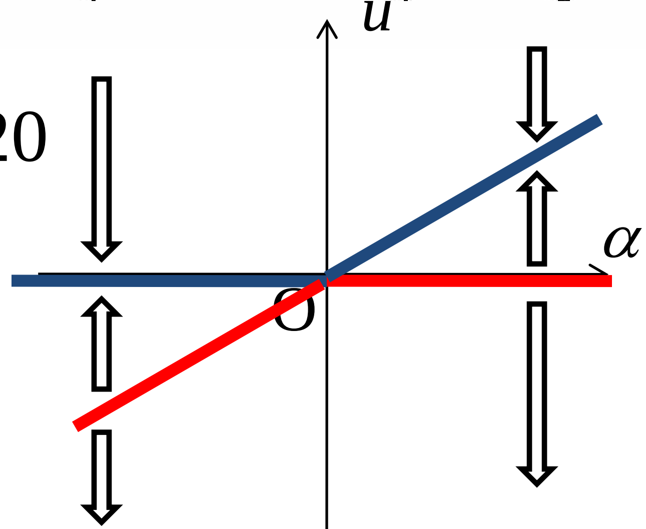




$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\alpha u + u^3$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kv$$

$k = -20$



$k = 20$

- ホップ分岐

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -v + \alpha u - \left(u^2 + v^2\right)\left(u - \beta v\right)$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + \alpha v - \left(u^2 + v^2\right)\left(v + \beta u\right)$$

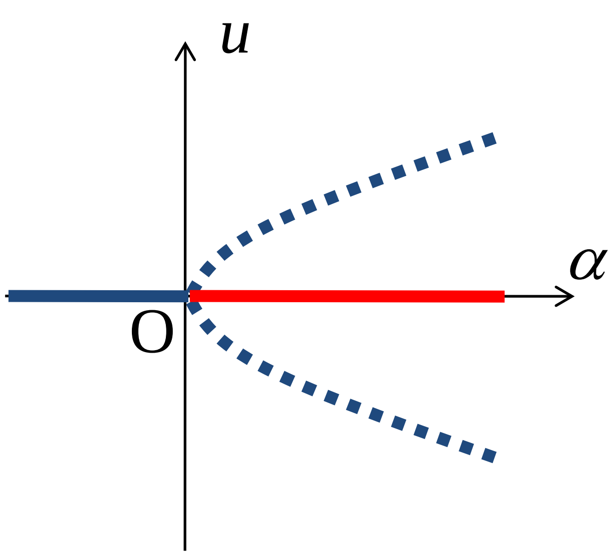
固定点：

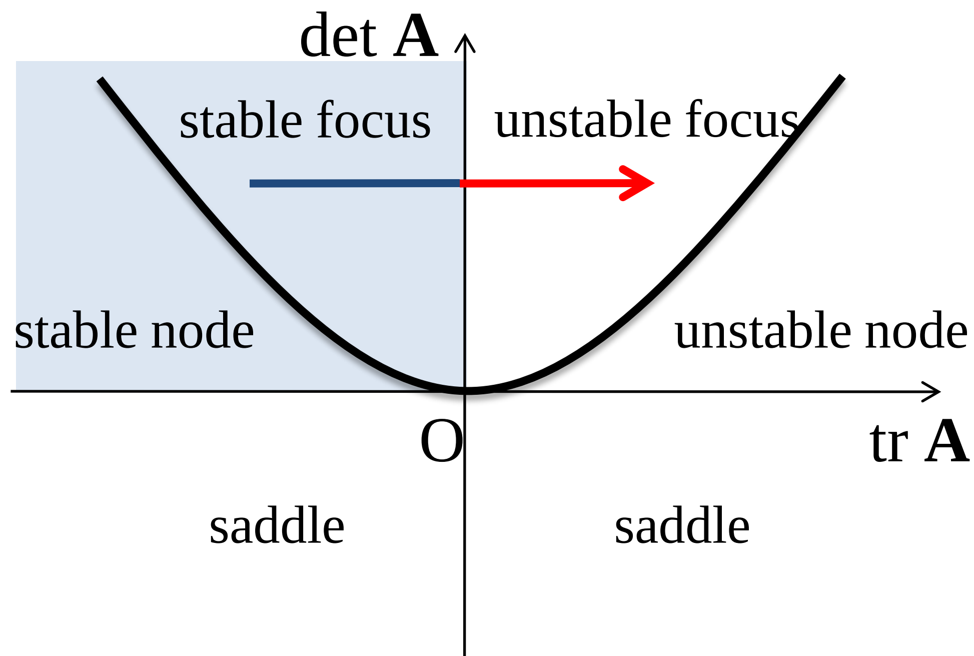
$$u = 0 \qquad v = 0$$

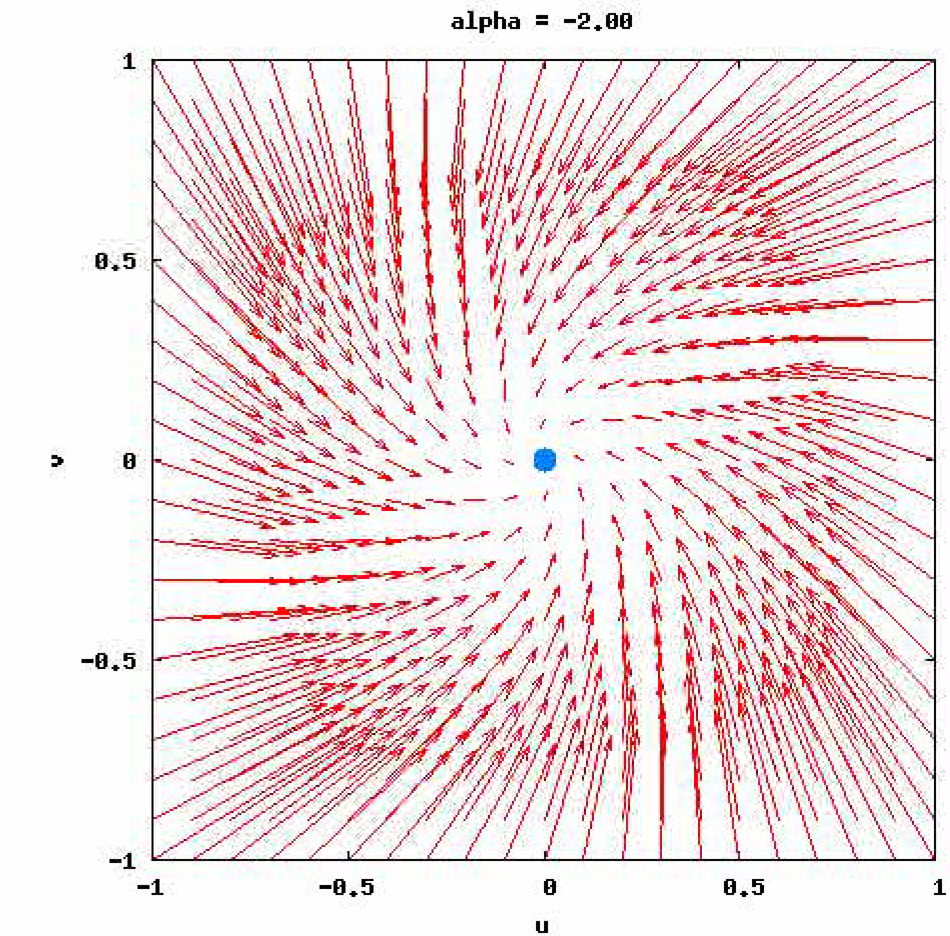
$u = 0, v = 0$ まわりで

$$\mathbf{A} = \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix}$$

$$\lambda = \alpha \pm i$$



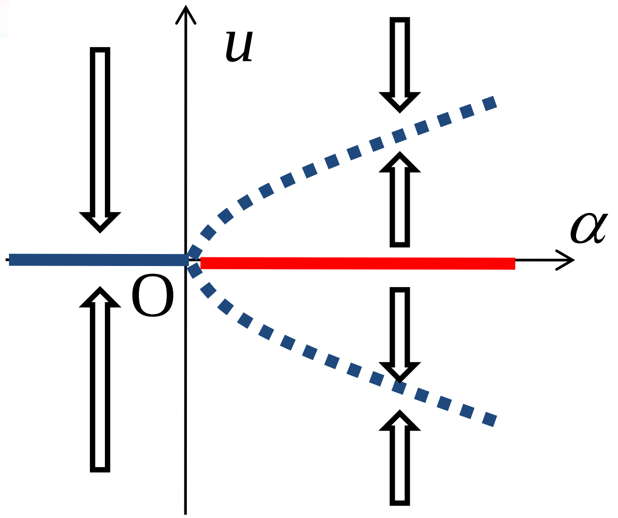




$\beta = 0$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -v + \alpha u - \left(u^2 + v^2\right)\left(u - \beta v\right)$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + \alpha v - \left(u^2 + v^2\right)\left(v + \beta u\right)$$



- ホップ分岐（別タイプ）

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -v - \alpha u + \left(u^2 + v^2\right)\left(u - \beta v\right)$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = u - \alpha v + \left(u^2 + v^2\right)\left(v + \beta u\right)$$

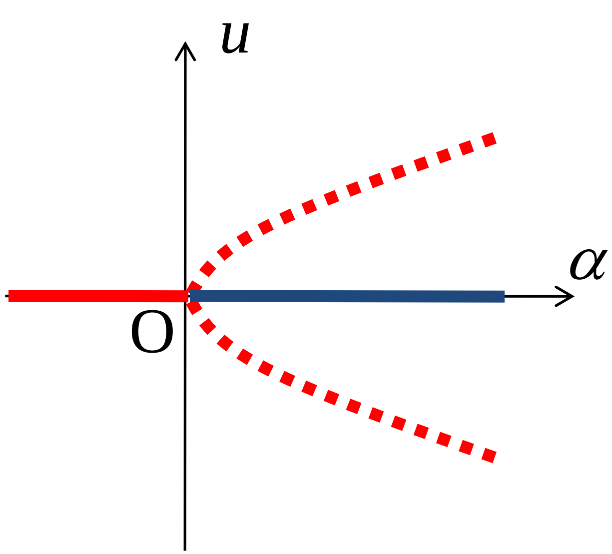
固定点：

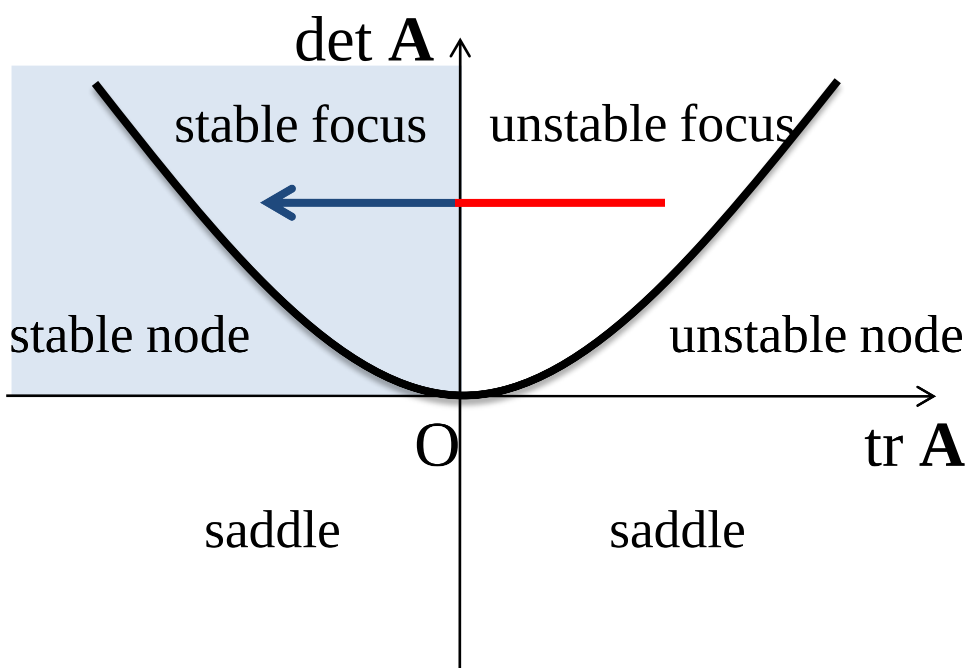
$$u = 0 \quad v = 0$$

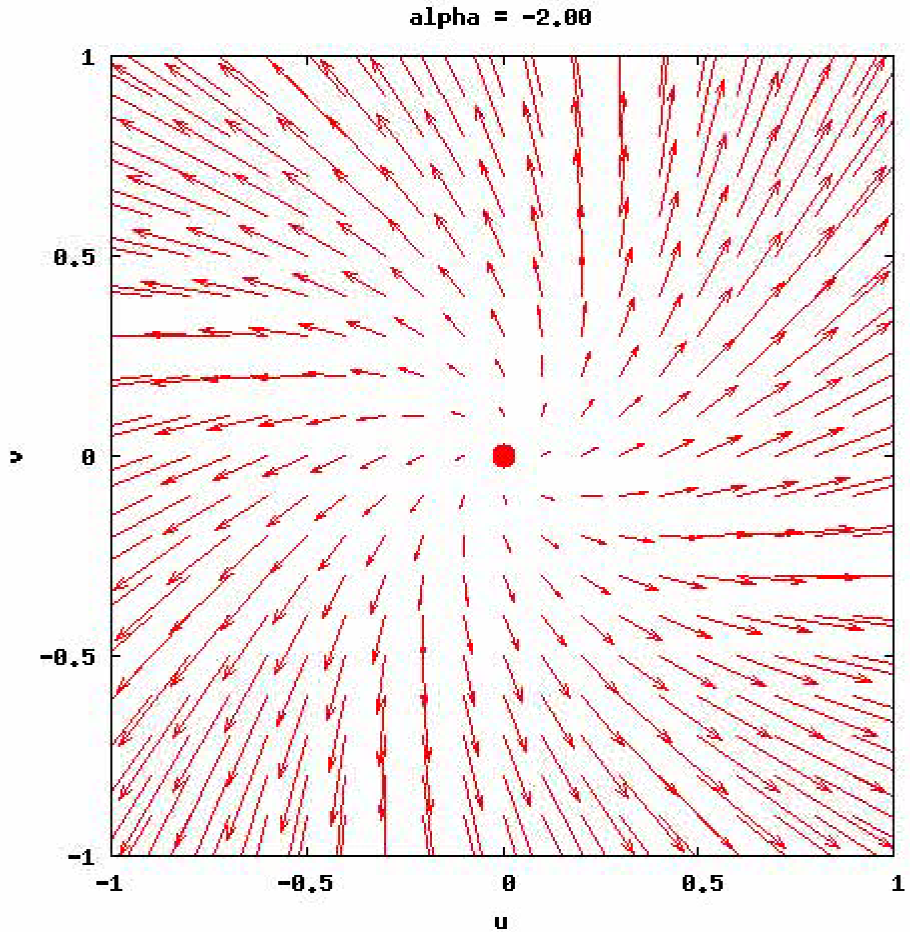
$u = 0, v = 0$ まわりで

$$\mathbf{A} = \begin{pmatrix} -\alpha & -1 \\ 1 & -\alpha \end{pmatrix}$$

$$\lambda = -\alpha \pm i$$



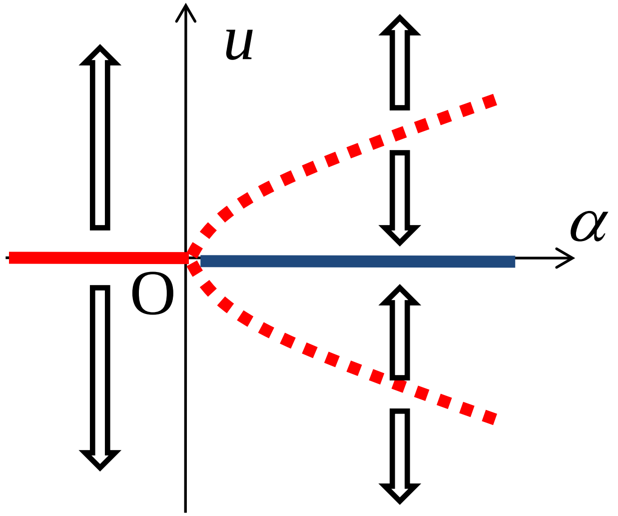




$\beta = 0$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -v - \alpha u + \left(u^2 + v^2\right)\left(u - \beta v\right)$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = u - \alpha v + \left(u^2 + v^2\right)\left(v + \beta u\right)$$



別の面からみた分岐の特徴付け

- スーパークリティカル（超臨界）分岐

- サブクリティカル（亜臨界）分岐