非線形振動子

2008.12.2

Stuart-Landau方程式

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = ax - y - \left(x^2 + y^2\right)\left(x - by\right) \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = ay + x - \left(x^2 + y^2\right)\left(y + bx\right) \end{cases}$$

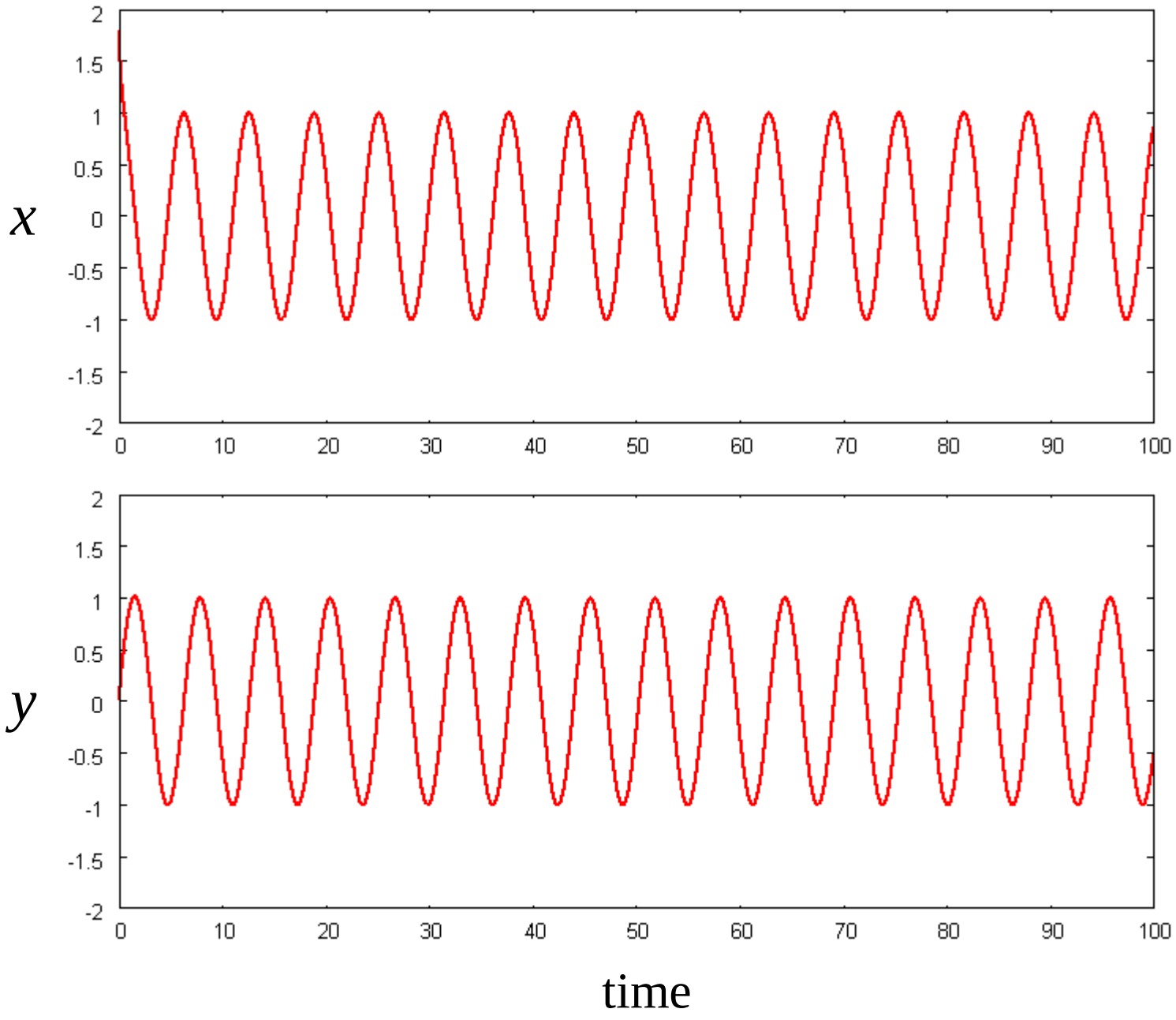
$$\begin{cases} \dfrac{\mathrm{d}r}{\mathrm{d}t} = ar - r^3 \\ \dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \omega \end{cases}$$

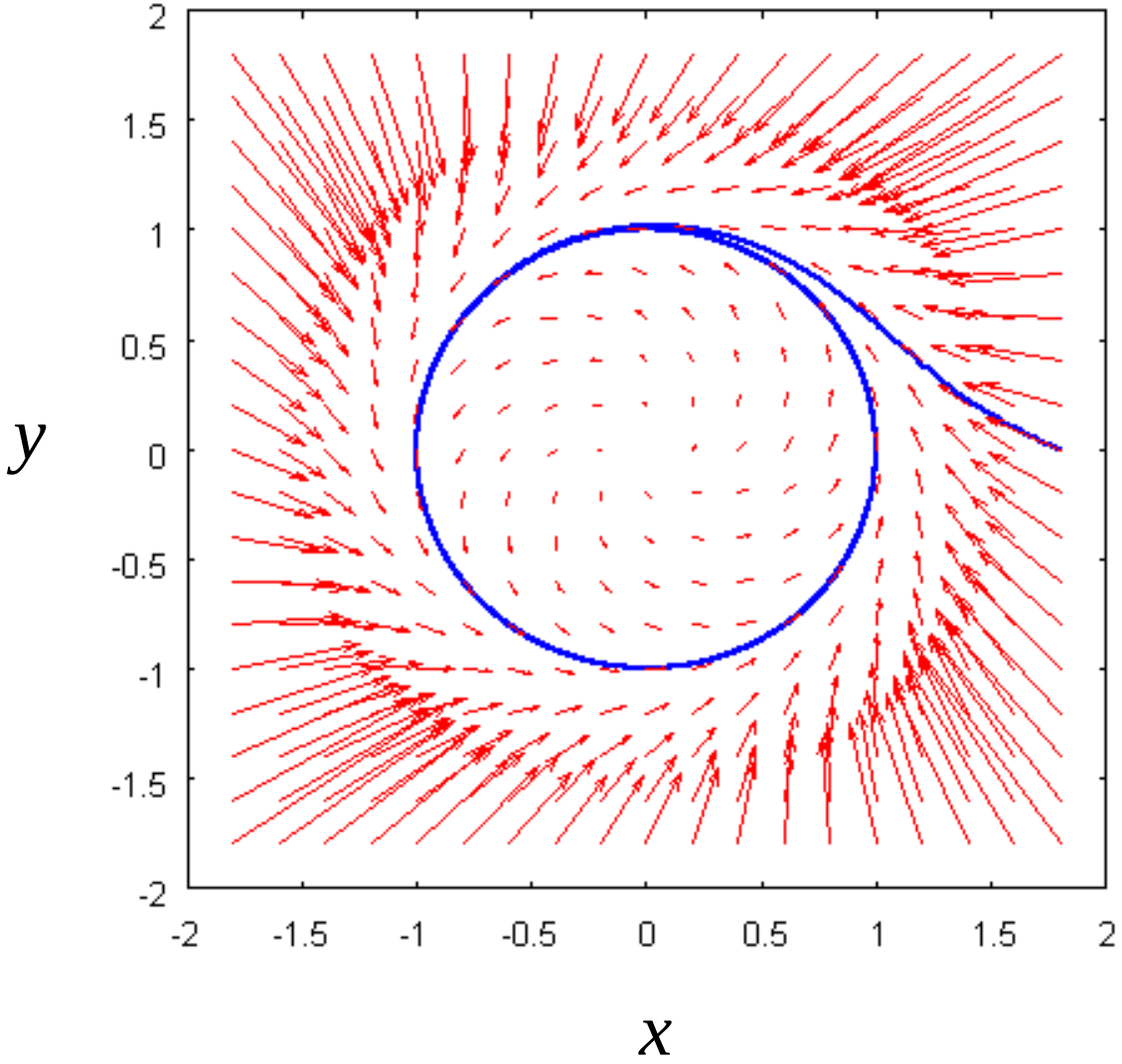
$$r^2 = x^2 + y^2$$

$$\frac{y}{x} = \tan\theta$$

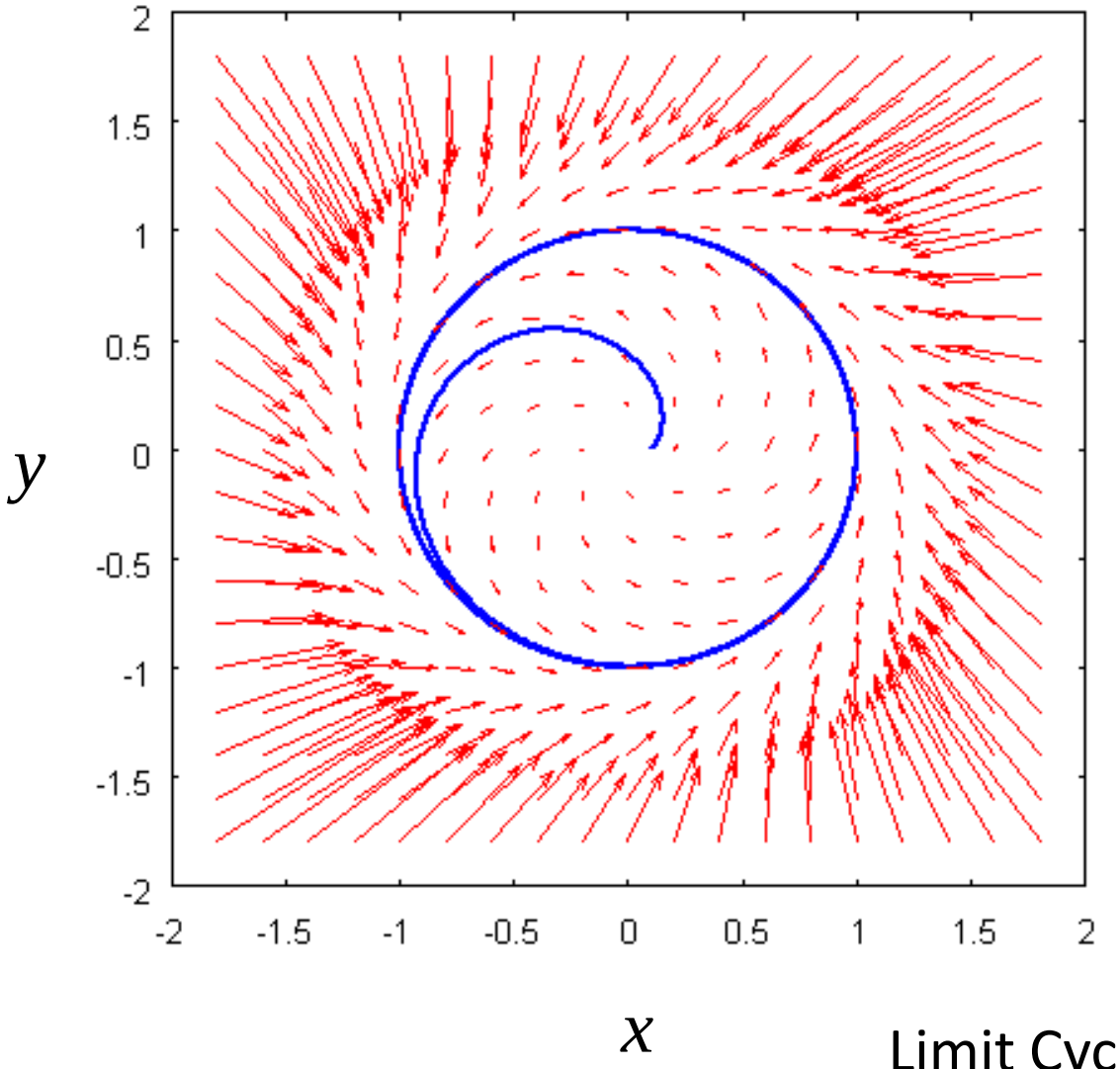
$$a = 1$$

$$b = 0$$


x
y
time
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
10
20
30
40
50
60
70
80
90
100

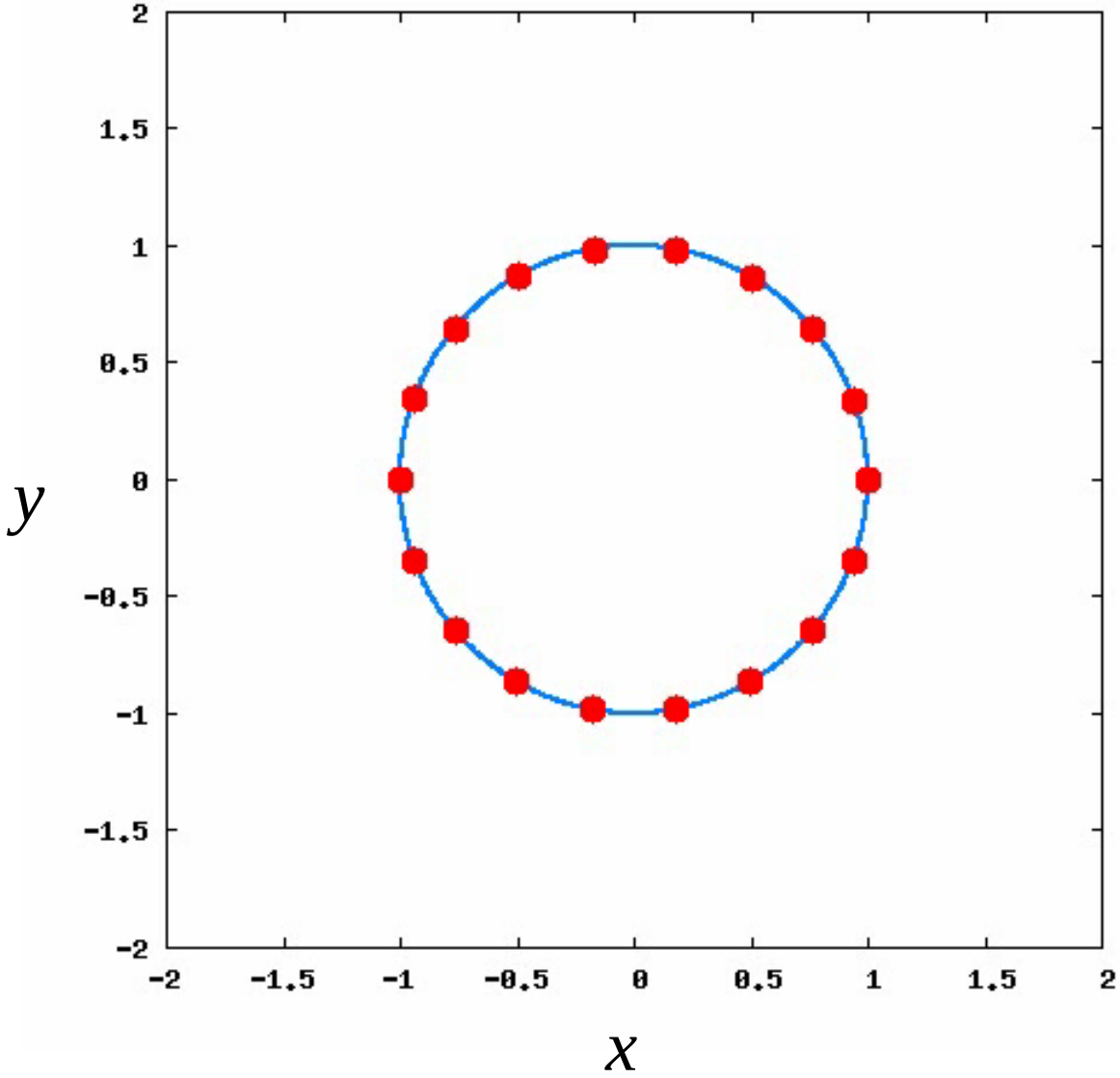

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
y
x

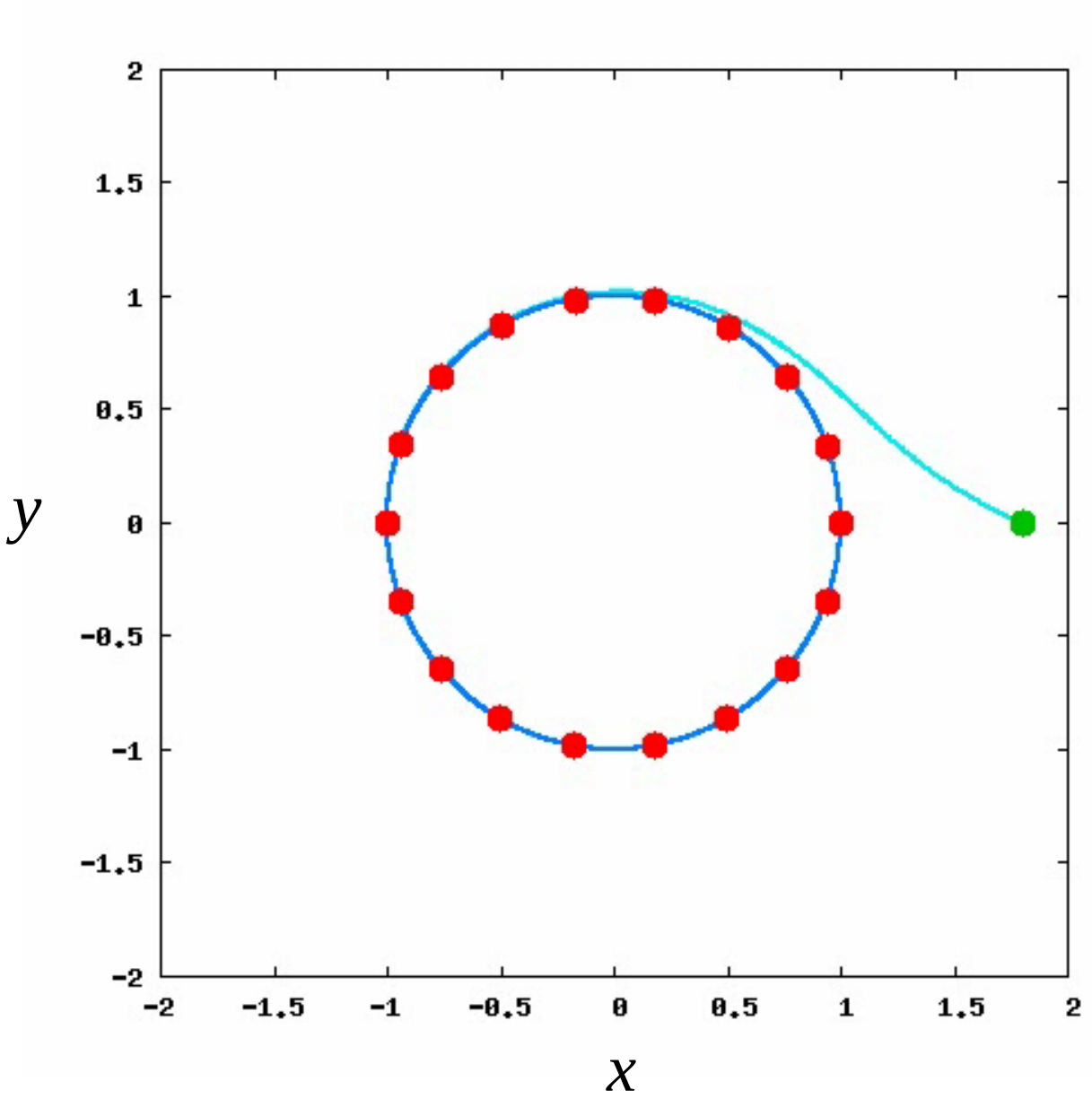
初期値を変えても

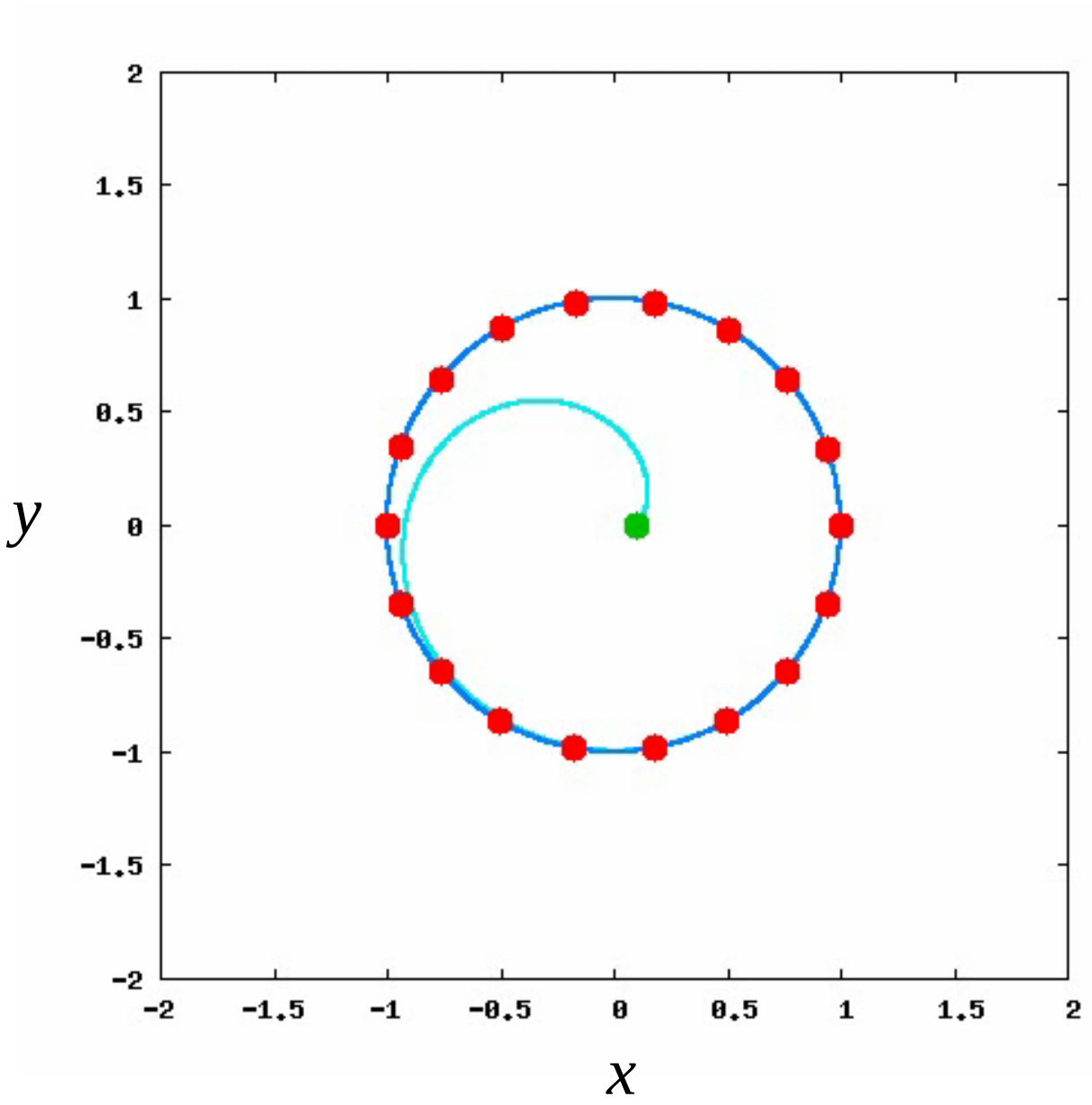


Limit Cycle (極限軌道)

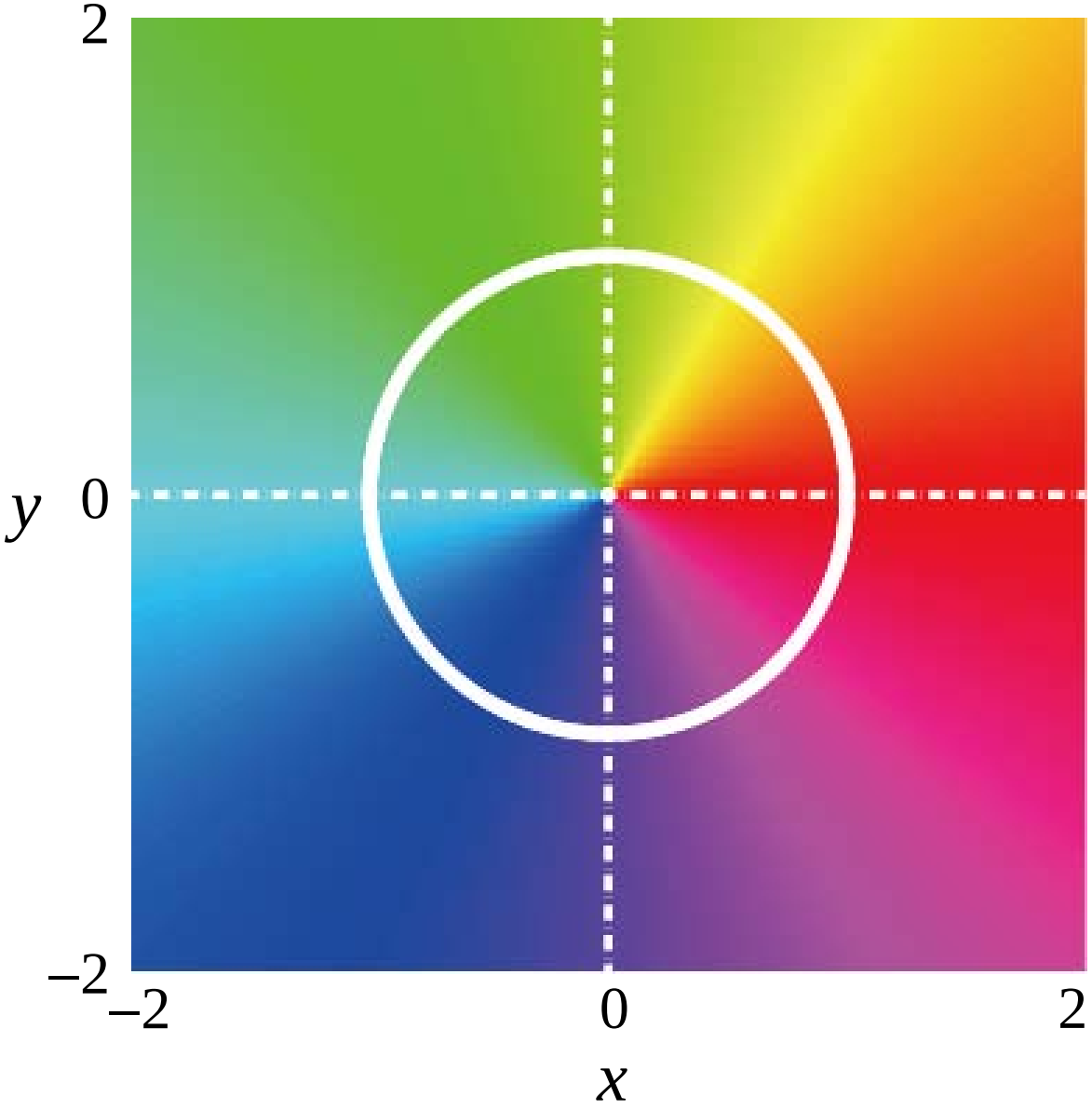
リミットサイクル上の運動




y
x
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
y
x

等位相面

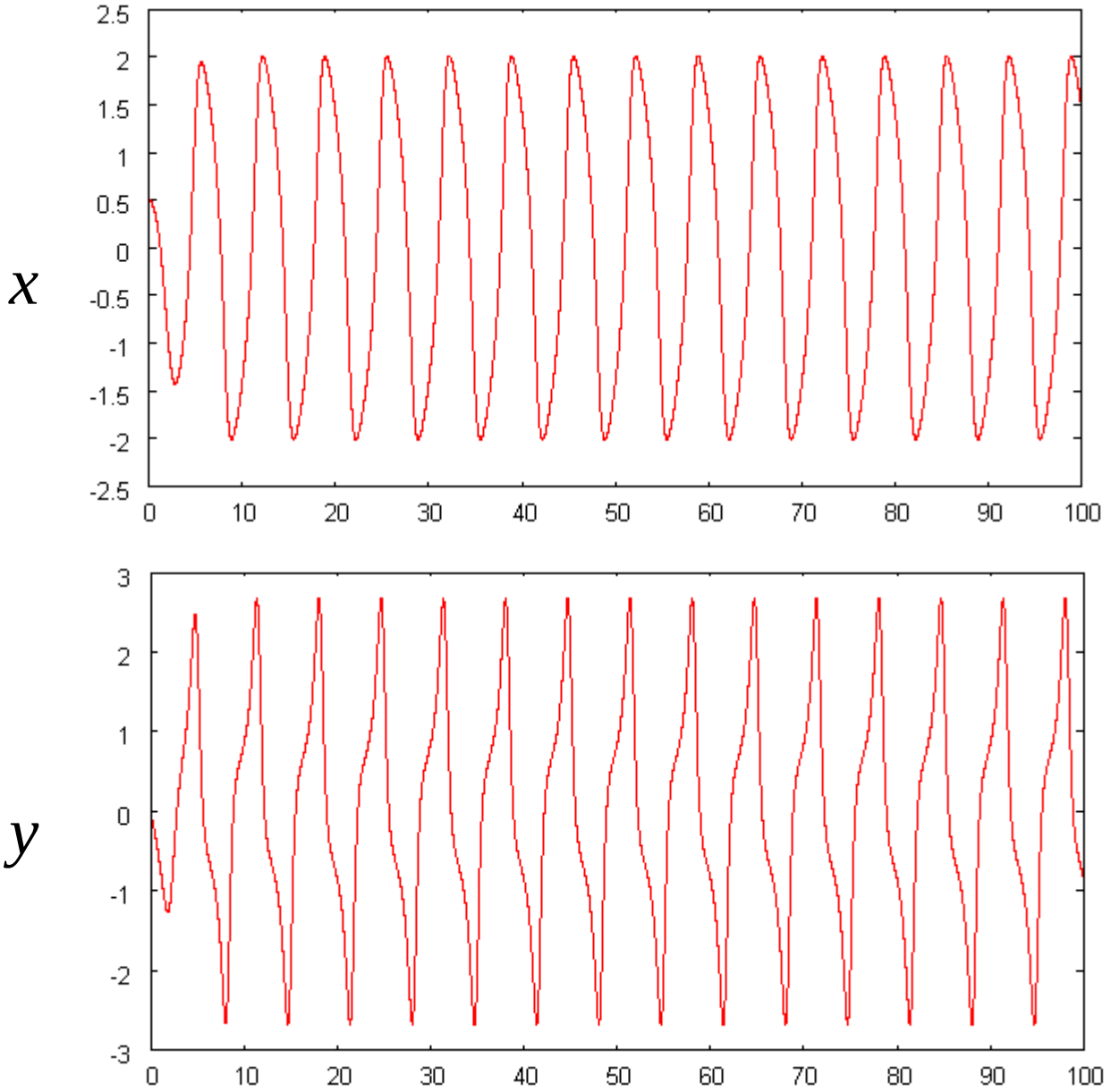


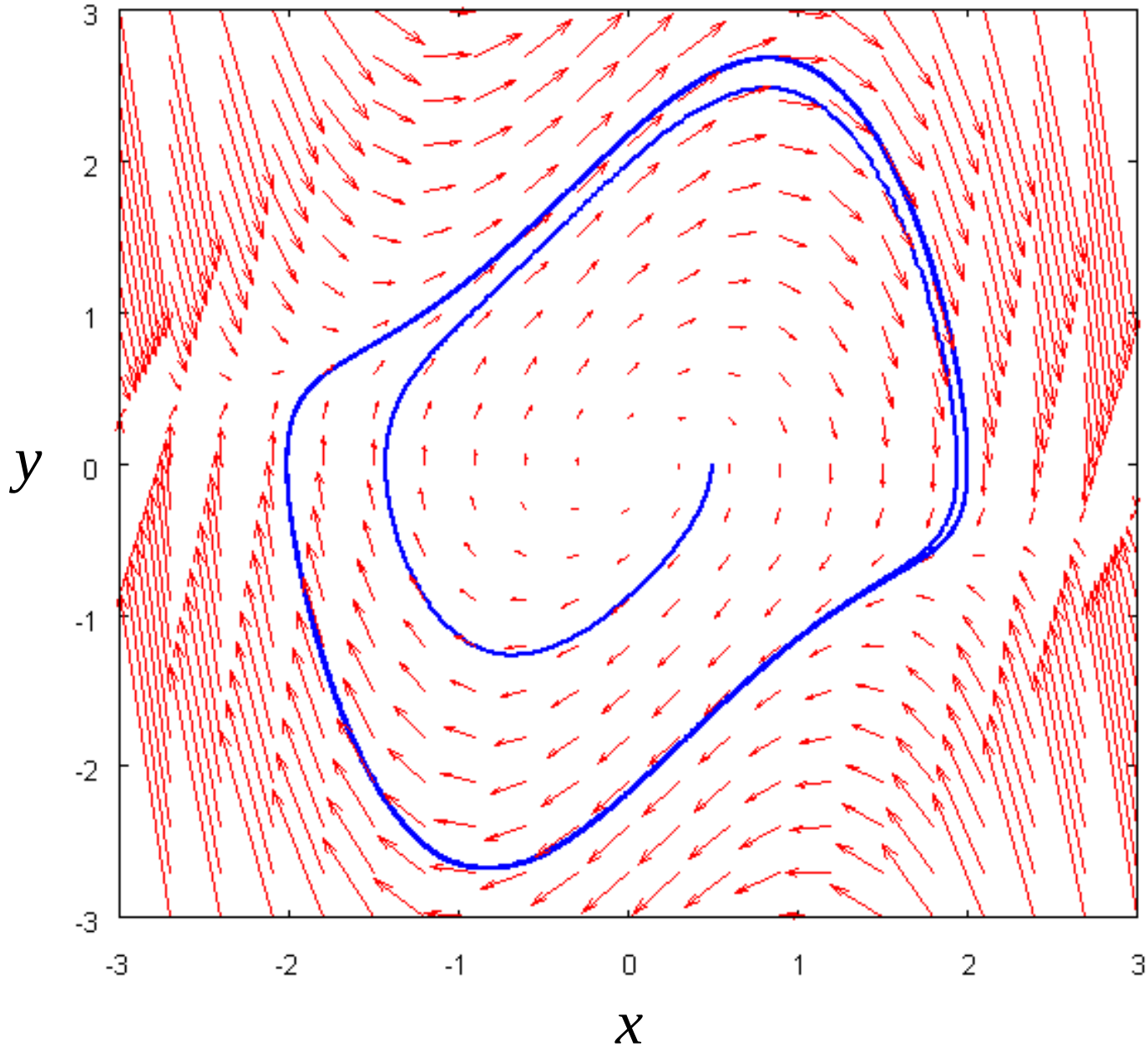
$$\theta = \arctan\left(\frac{y}{x}\right)$$

van der Pol 方程式　～　丸くなくても・・・

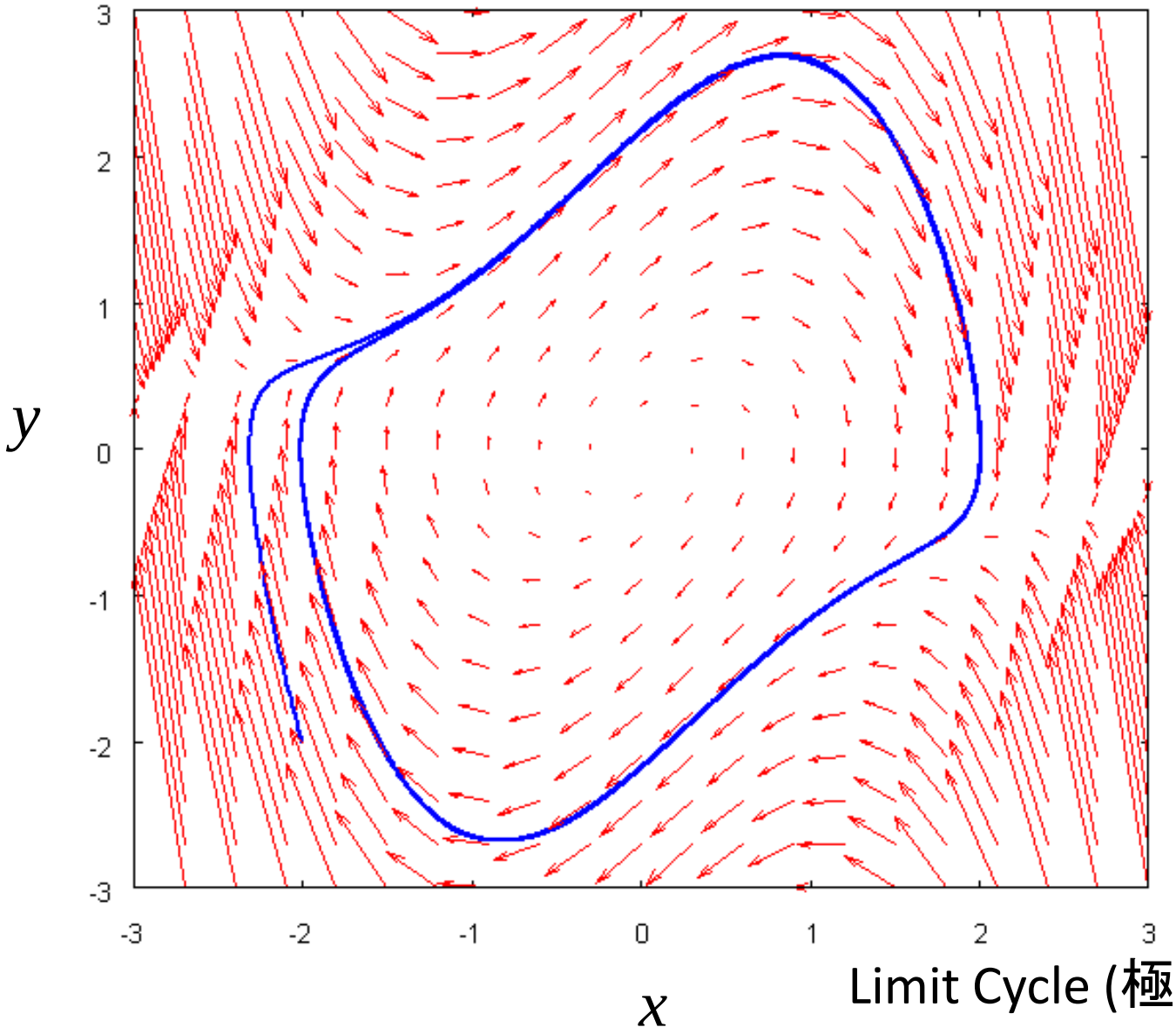
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \alpha\left(x^2 - 1\right)\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0$$

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = y \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -\alpha\left(x^2 - 1\right)y - x \end{cases}$$

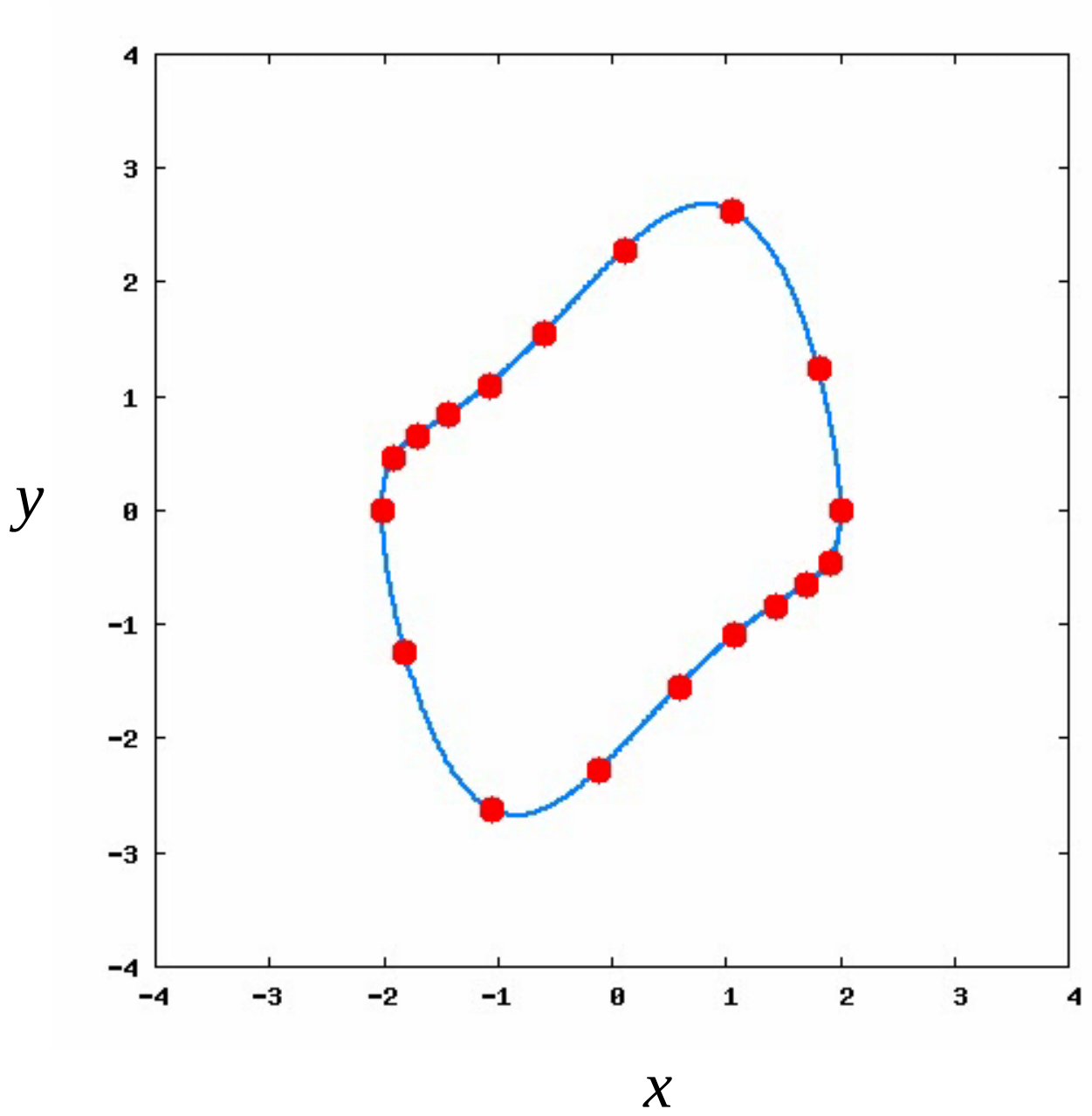

x
y
time
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
3
2
1
0
-1
-2
-3
0
10
20
30
40
50
60
70
80
90
100

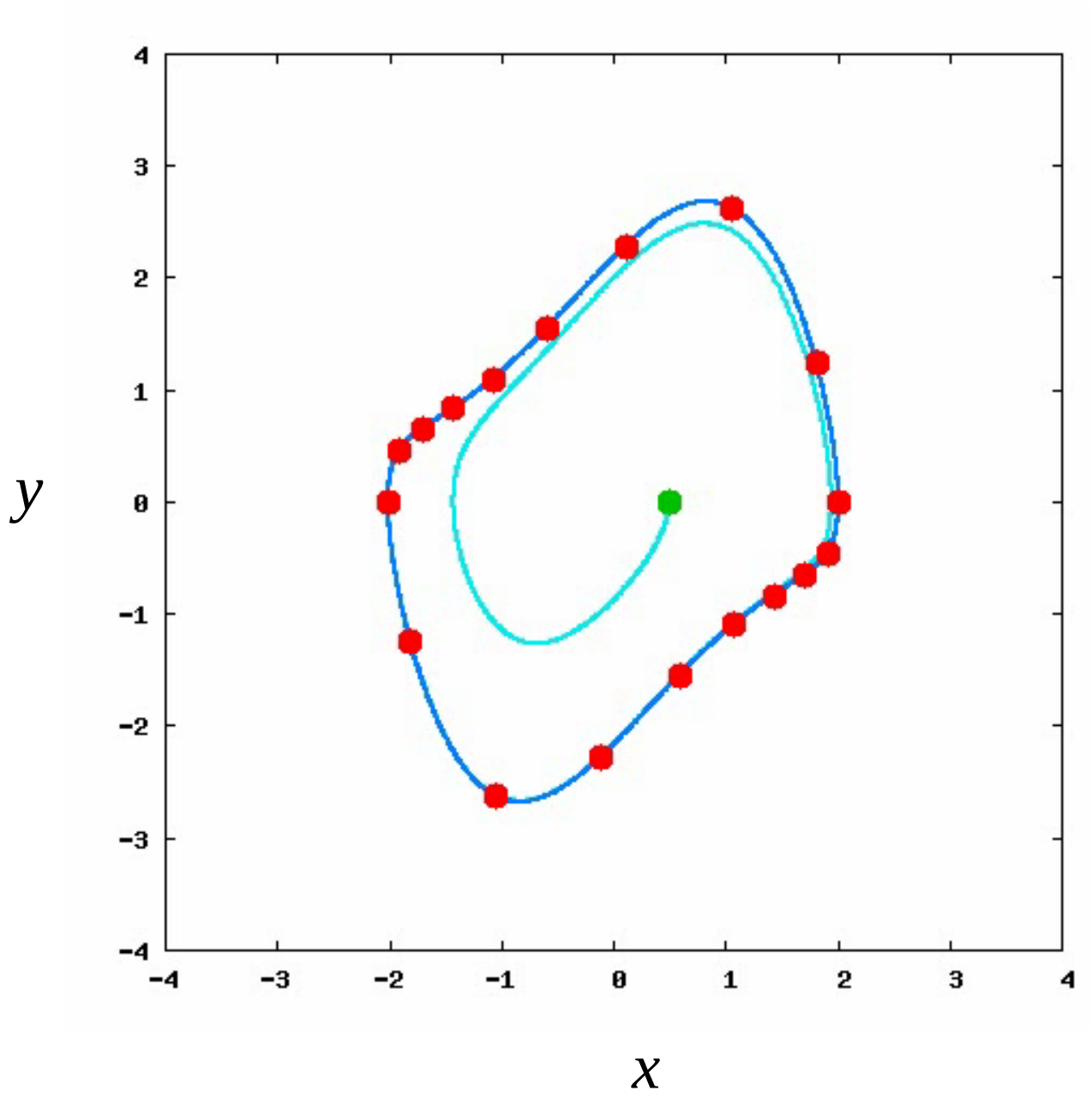

3
2
1
0
-1
-2
-3
y
-3
-2
-1
0
1
2
3
x

初期値を変えても

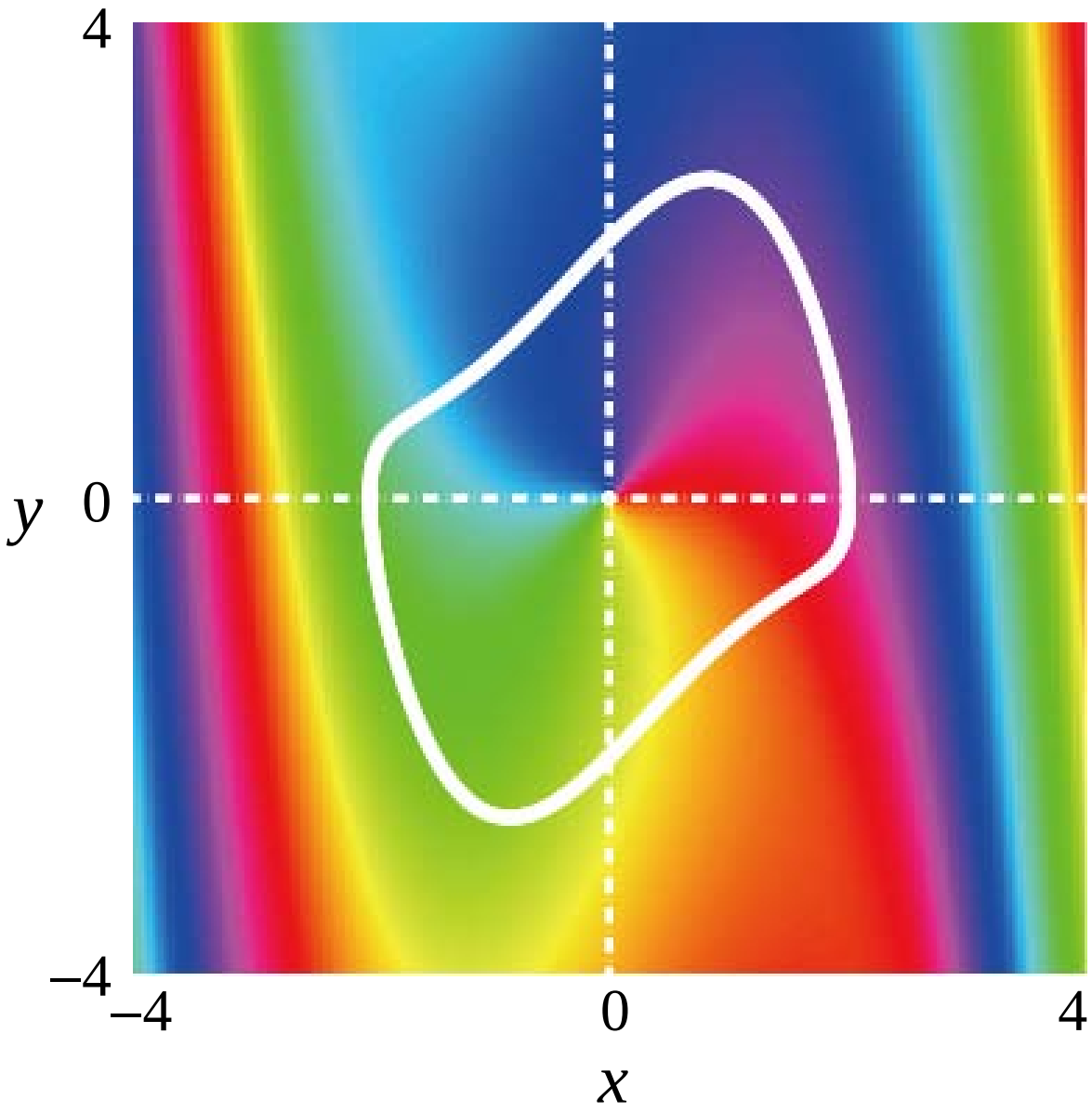


Limit Cycle (極限軌道)


y
x
4
3
2
1
0
−1
−2
−3
−4
−4
−3
−2
−1
0
1
2
3
4


y
x
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

等位相面



さまざまな非線形振動子

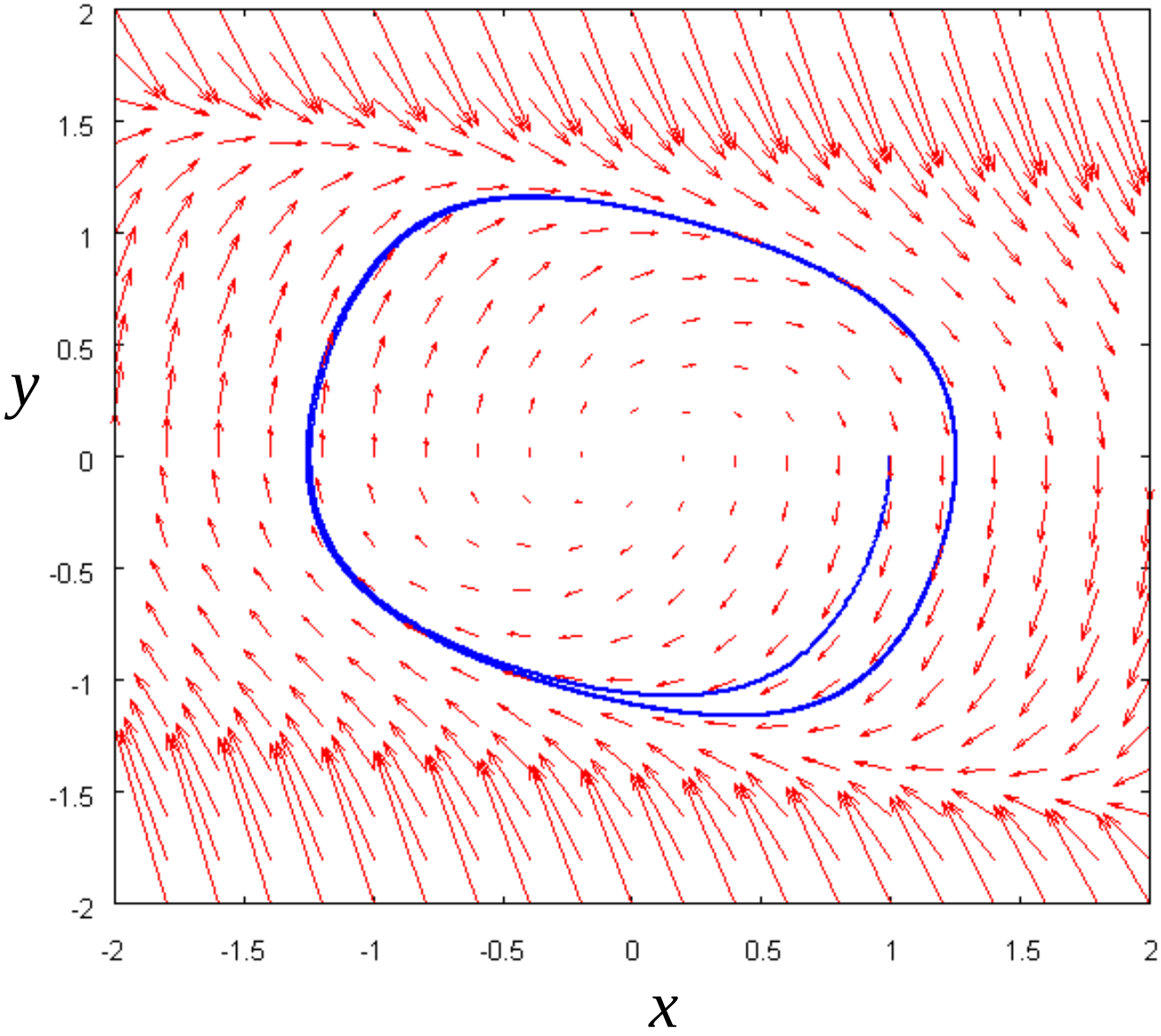
Rayleigh方程式

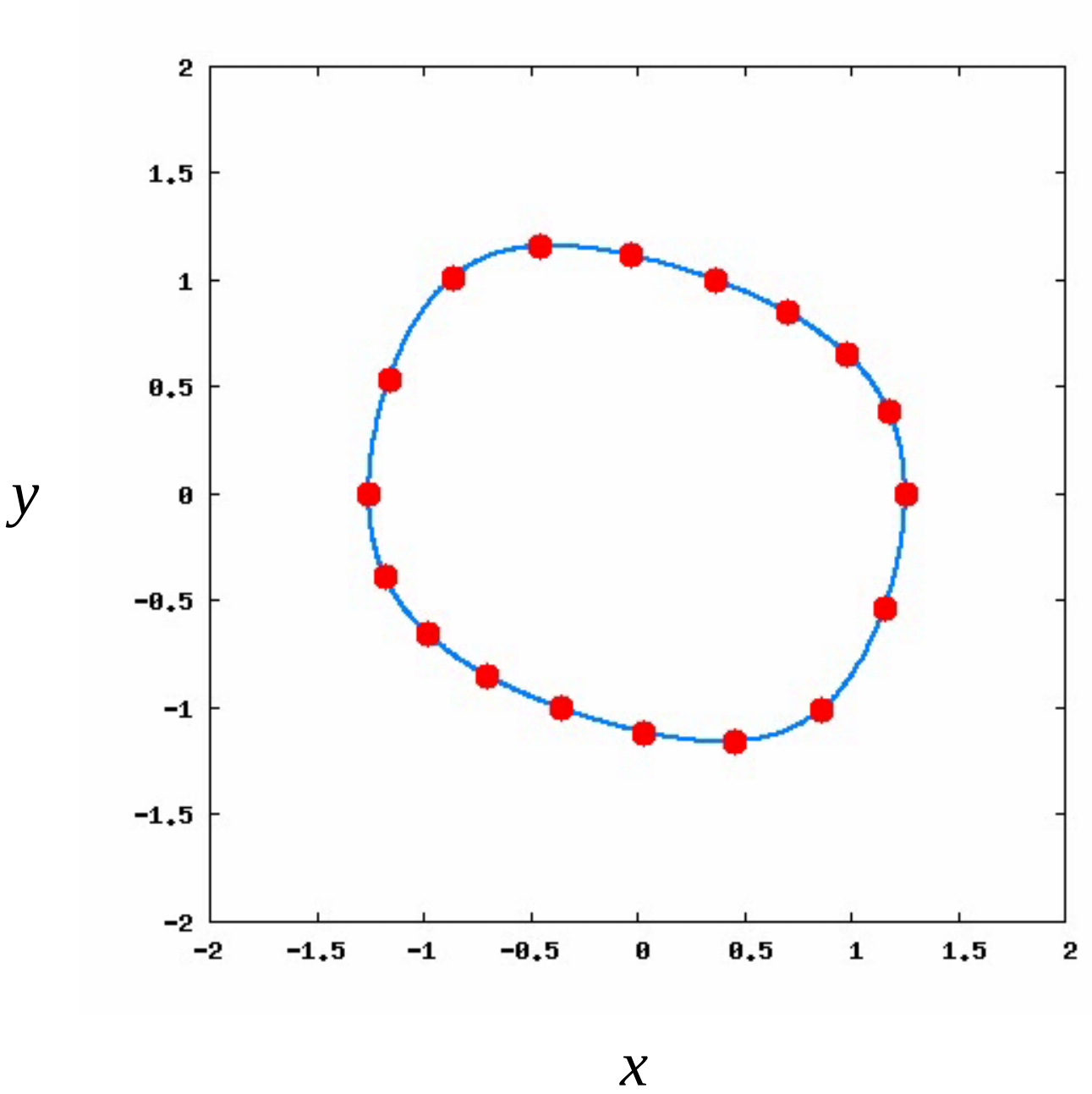
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \alpha\left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 - 1\right)\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0$$

FitzHugh-Nagumo方程式

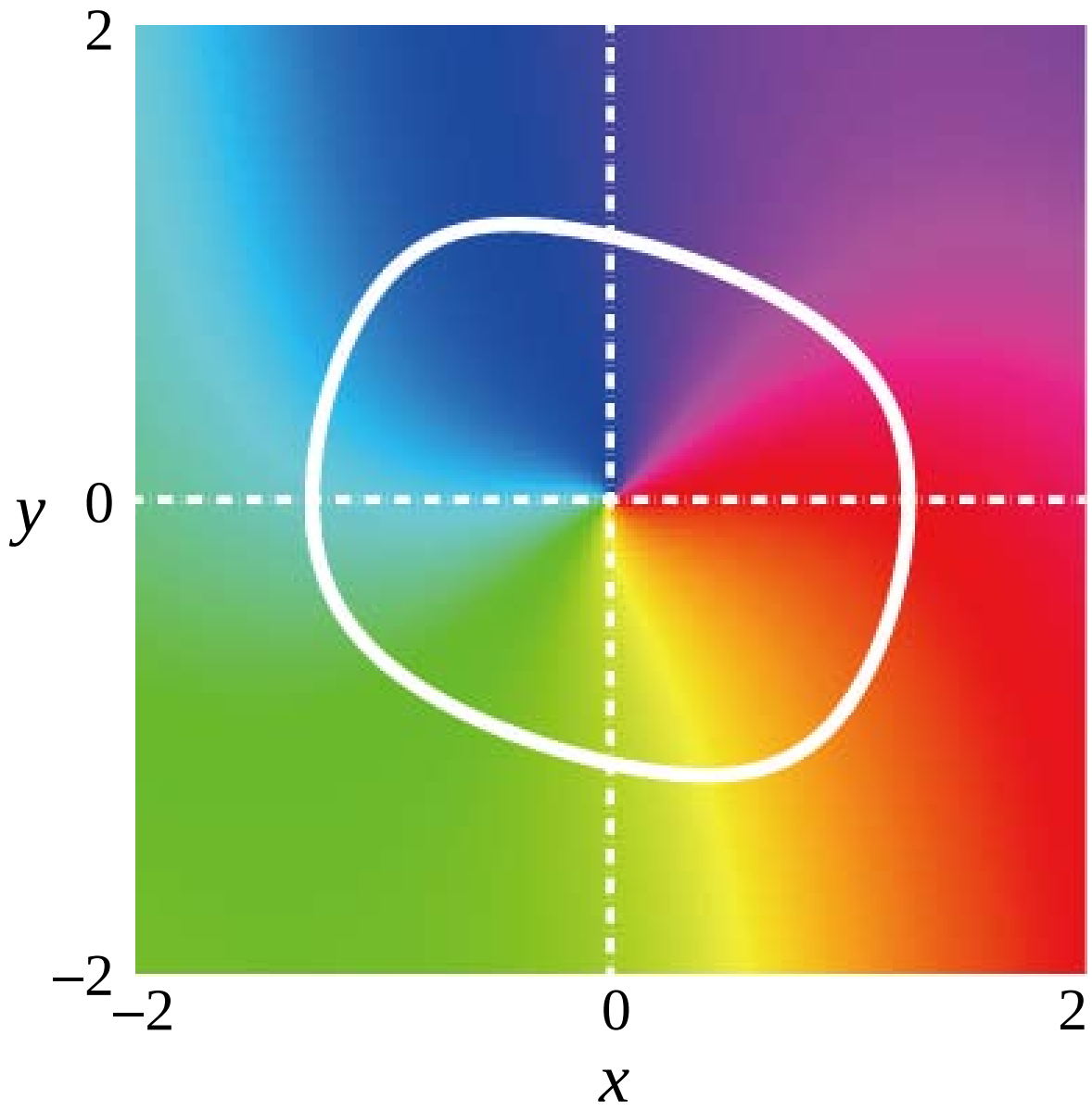
$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1}{\varepsilon}\left(x - x^3 - y\right) \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = x - y + b \end{cases}$$

Rayleigh方程式

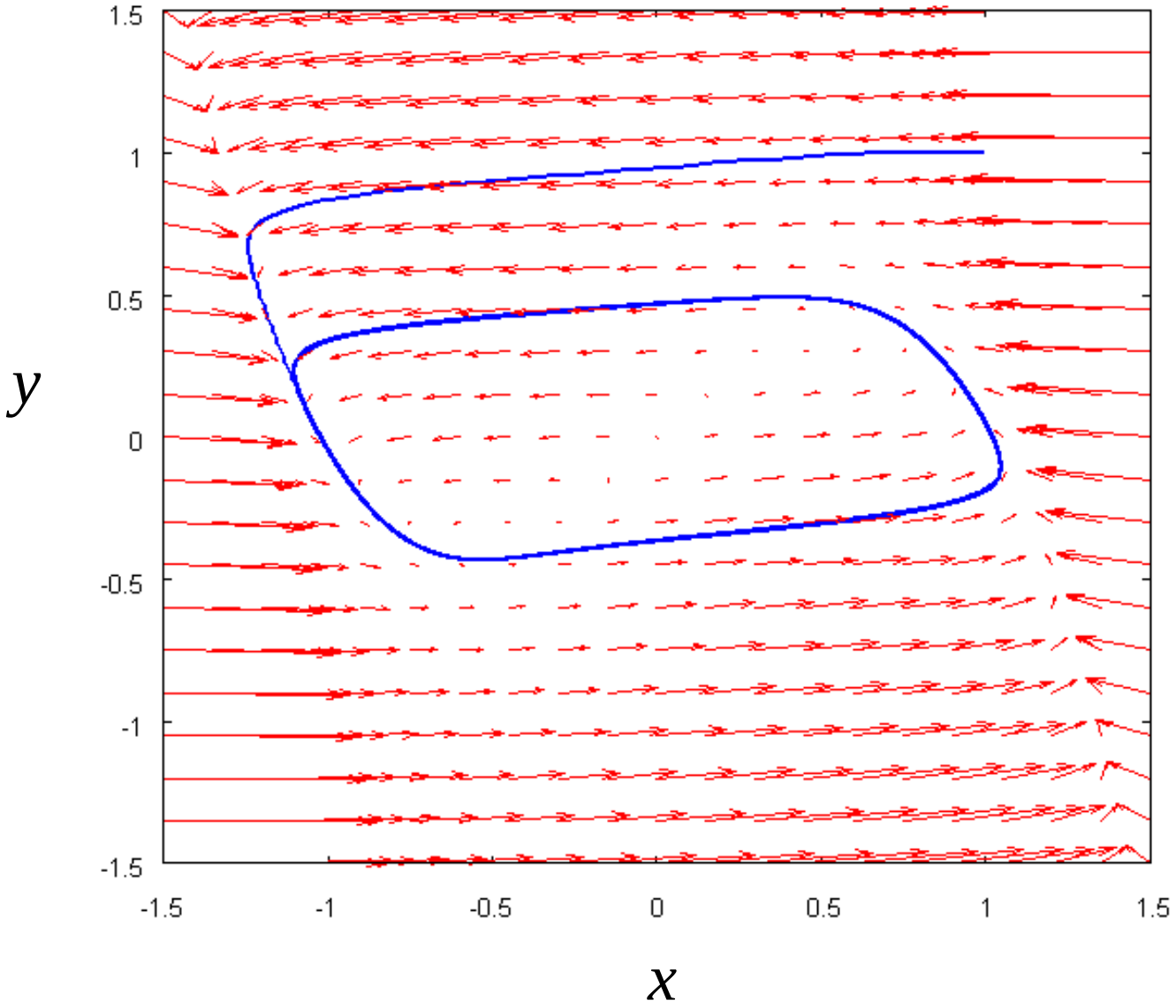


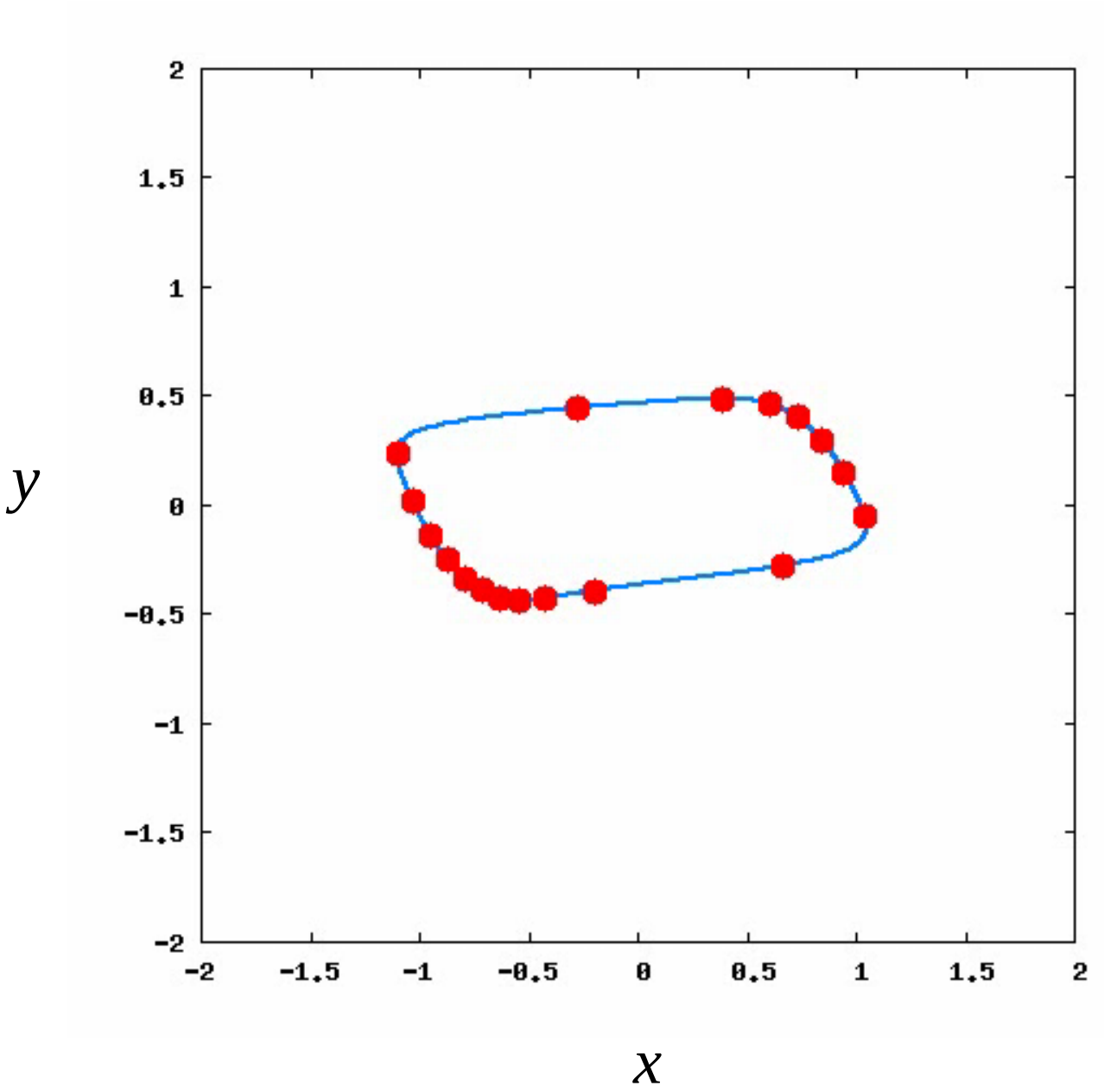

y
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x

等位相面



FitzHugh-Nagumo方程式




y
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x

等位相面