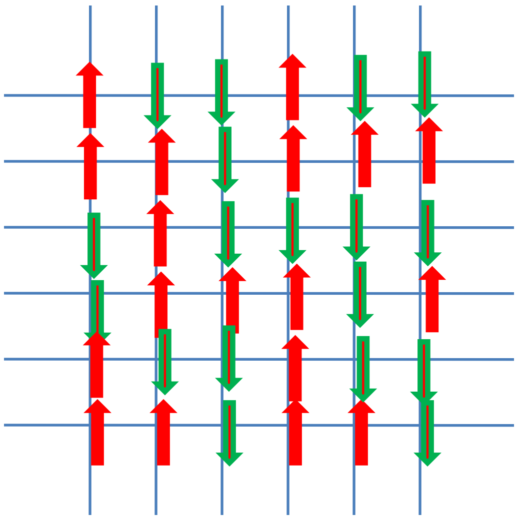
2008.12.9

反応拡散系とパターン形成

スピン系での局所ポテンシャル



上向きスピンの割合:　ϕ

サイト数:　N

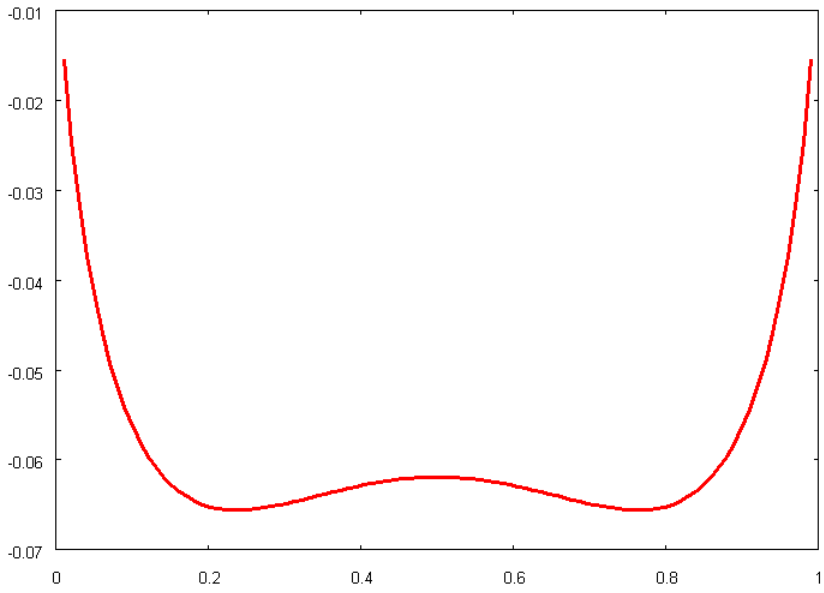
最近接サイト数:　a

相互作用エネルギーの差:　ε

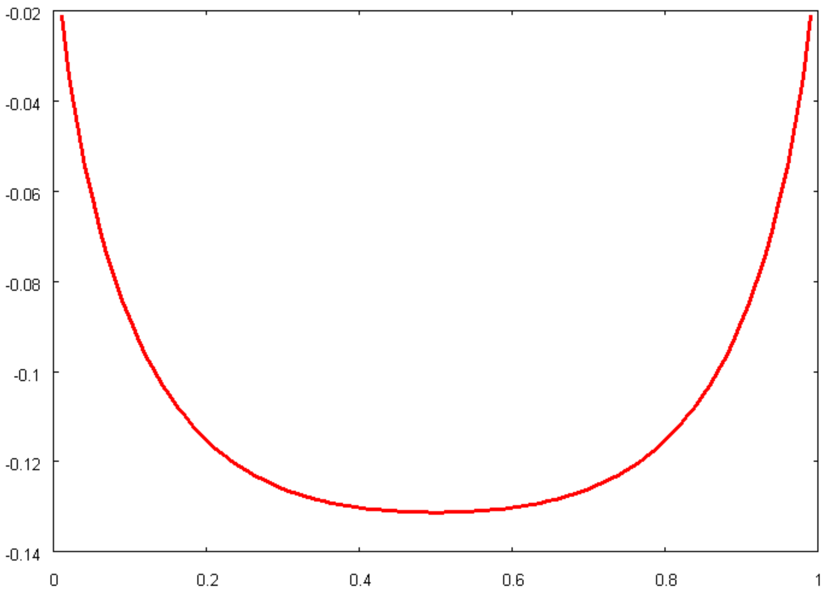
$$F = \frac{Na\varepsilon}{2}\phi(1-\phi) - Nk_{\mathrm{B}}T\left[\phi\ln\phi + (1-\phi)\ln(1-\phi)\right]$$

ポテンシャルの形状

$T < T_c$

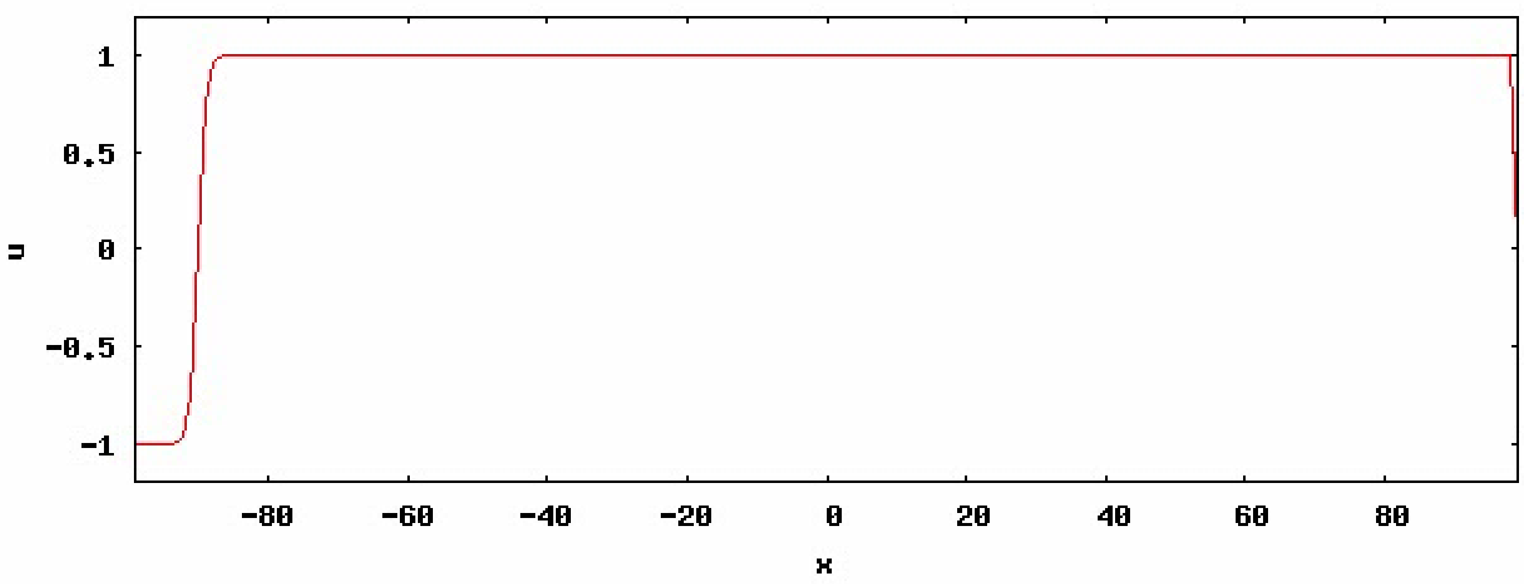


$T > T_c$

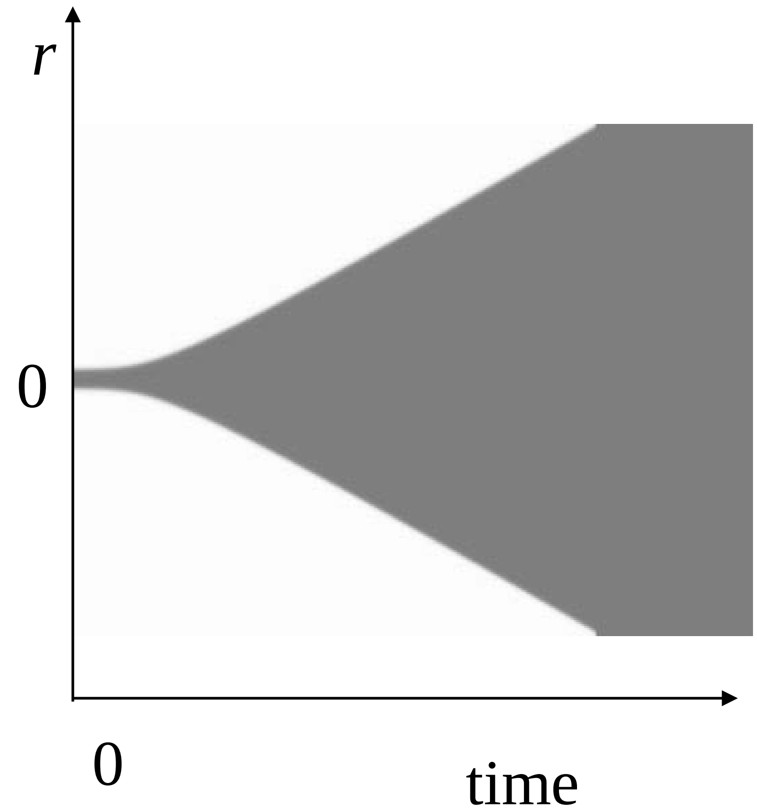


1次元系での伝播

$$\frac{\partial u}{\partial t} = K\left[u - u^3 - P + \varepsilon\nabla^2 u\right]$$



$$\frac{\partial u}{\partial t} = K\left[u - u^3 - P + \varepsilon \nabla^2 u\right]$$



Turingパターン（静止パターン）

$$\frac{\partial u}{\partial t} = -u^3 + u - 4v + D_u \nabla^2 u$$

$$\frac{\partial v}{\partial t} = u - 3v - a + D_v \nabla^2 v$$

$a = 0$では$u = v = 0$が固定点

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix}$$

tr A = –2、det A = 1より安定

$$u = 0 + \int \mathrm{d}k \ \Delta u(k) e^{ikx}$$

$$v = 0 + \int \mathrm{d}k \ \Delta v(k) e^{ikx}$$ とおいて

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = \begin{pmatrix} 1 - D_u k^2 & -4 \\ 1 & -3 - D_u k^2 \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix}$$

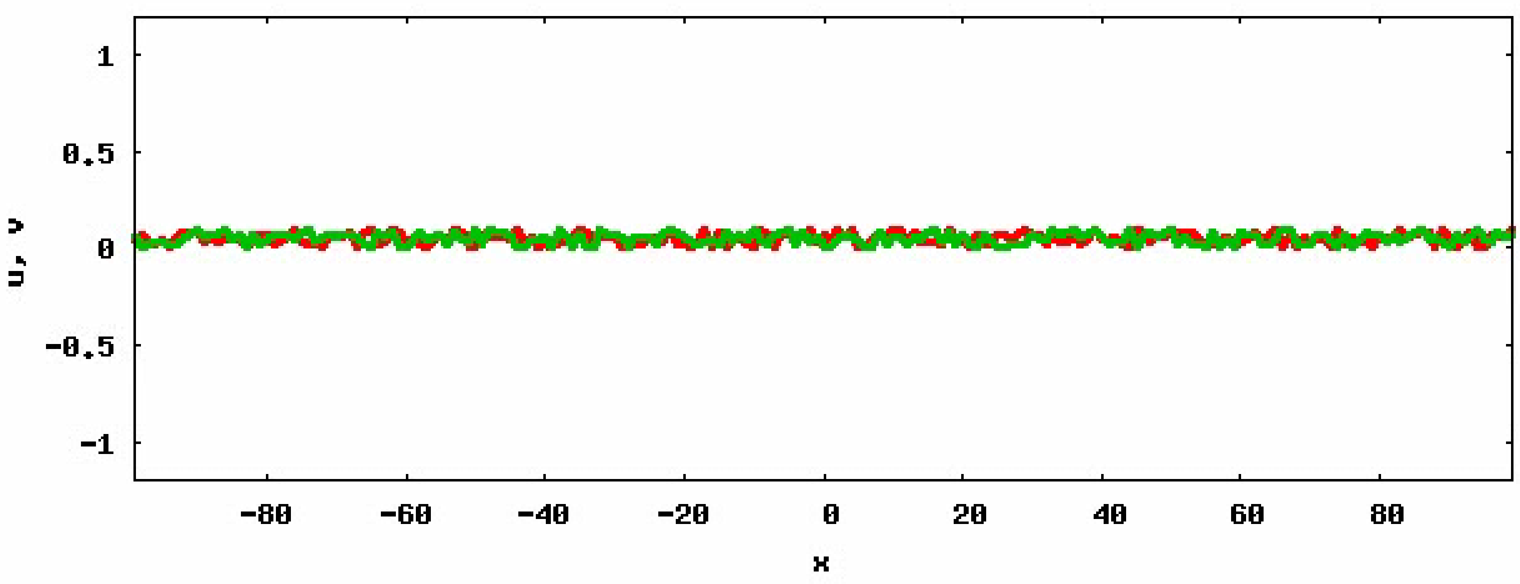
不安定化する条件は

$$D_v > 9D_u$$

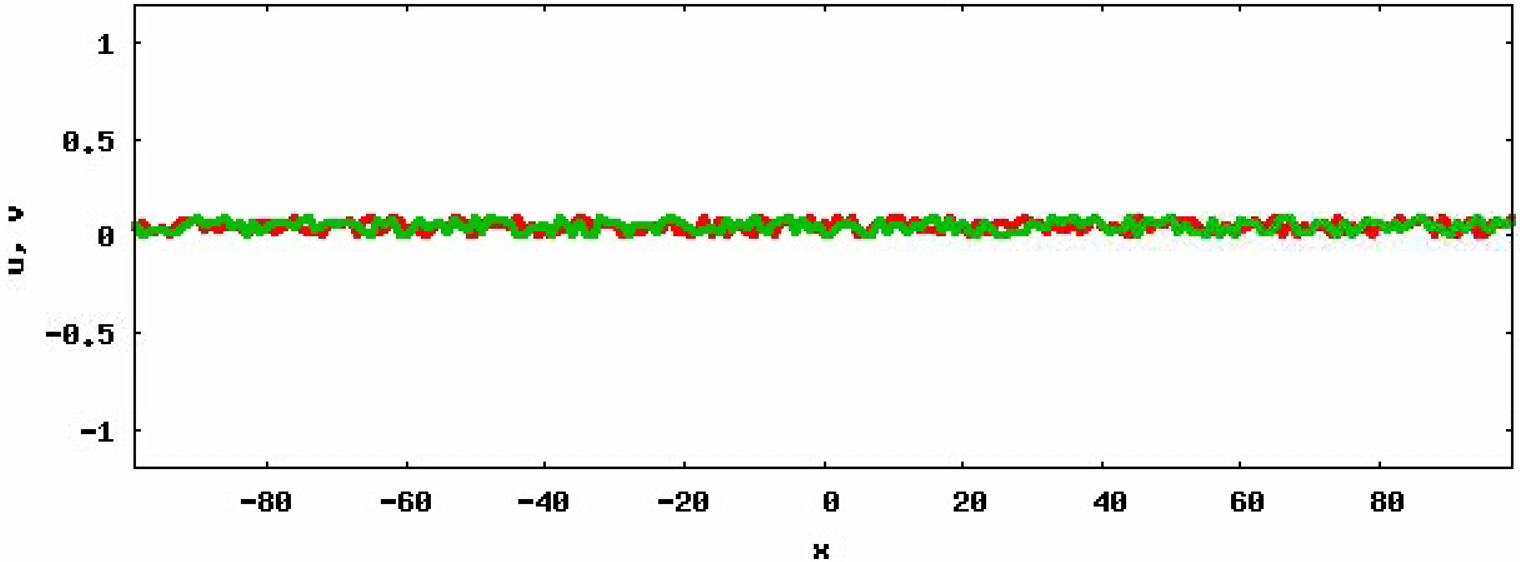
はじめに不安定化する波数は $k = \sqrt{\frac{1}{2D_u} - \frac{3}{2D_v}}$

1次元では

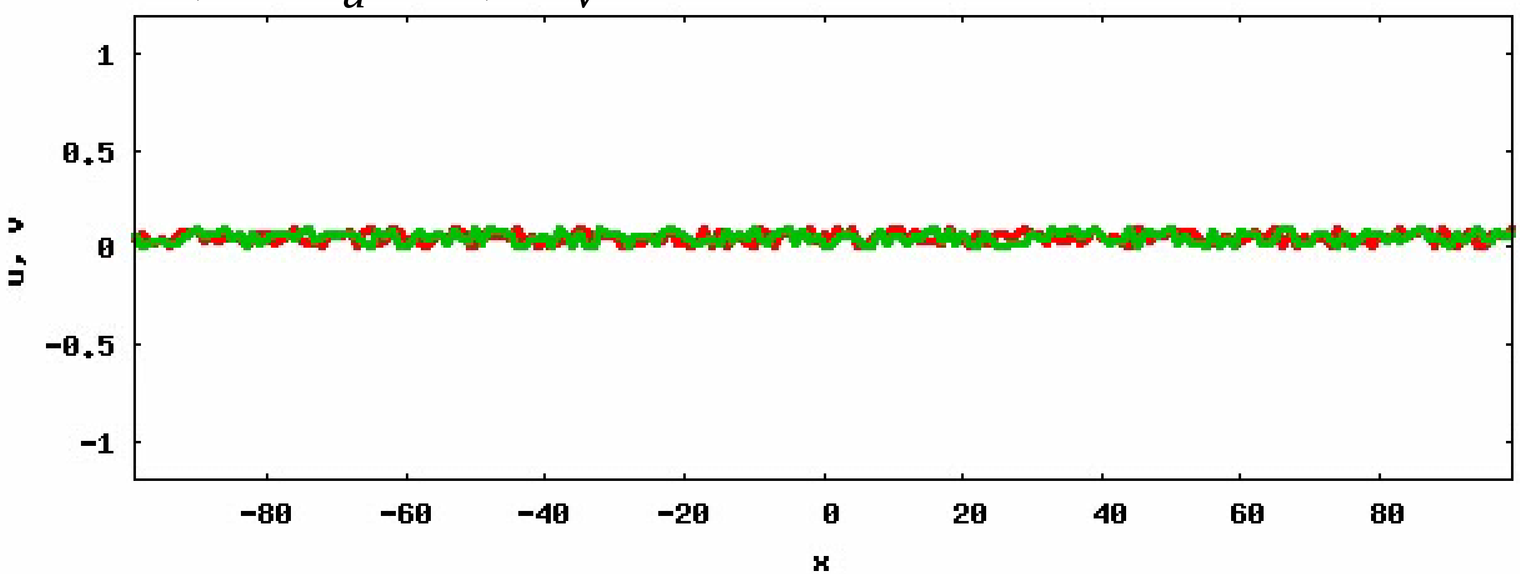
$a = 0, \quad D_u = 1, D_v = 5$



$a = 0, \quad D_u = 1, D_v = 20$

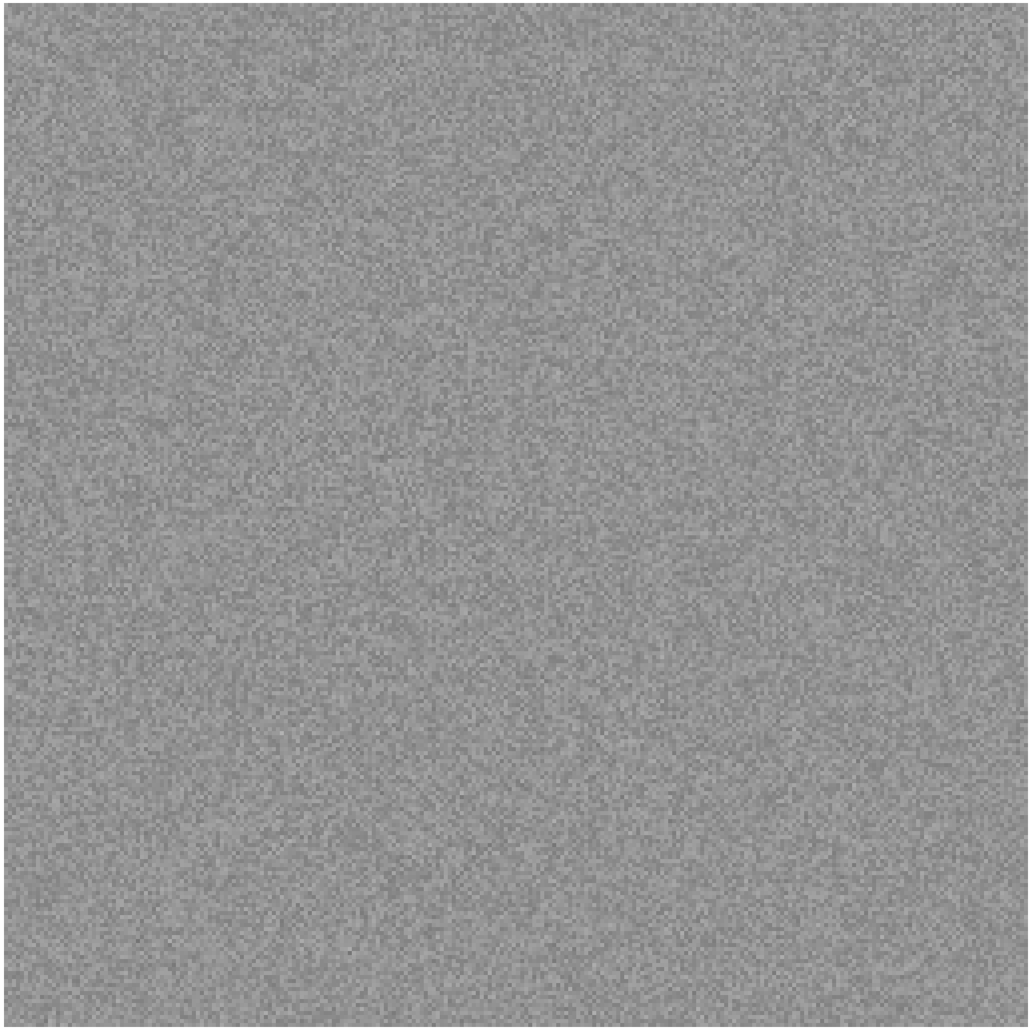


$a = 0, \quad D_u = 2, D_v = 20$

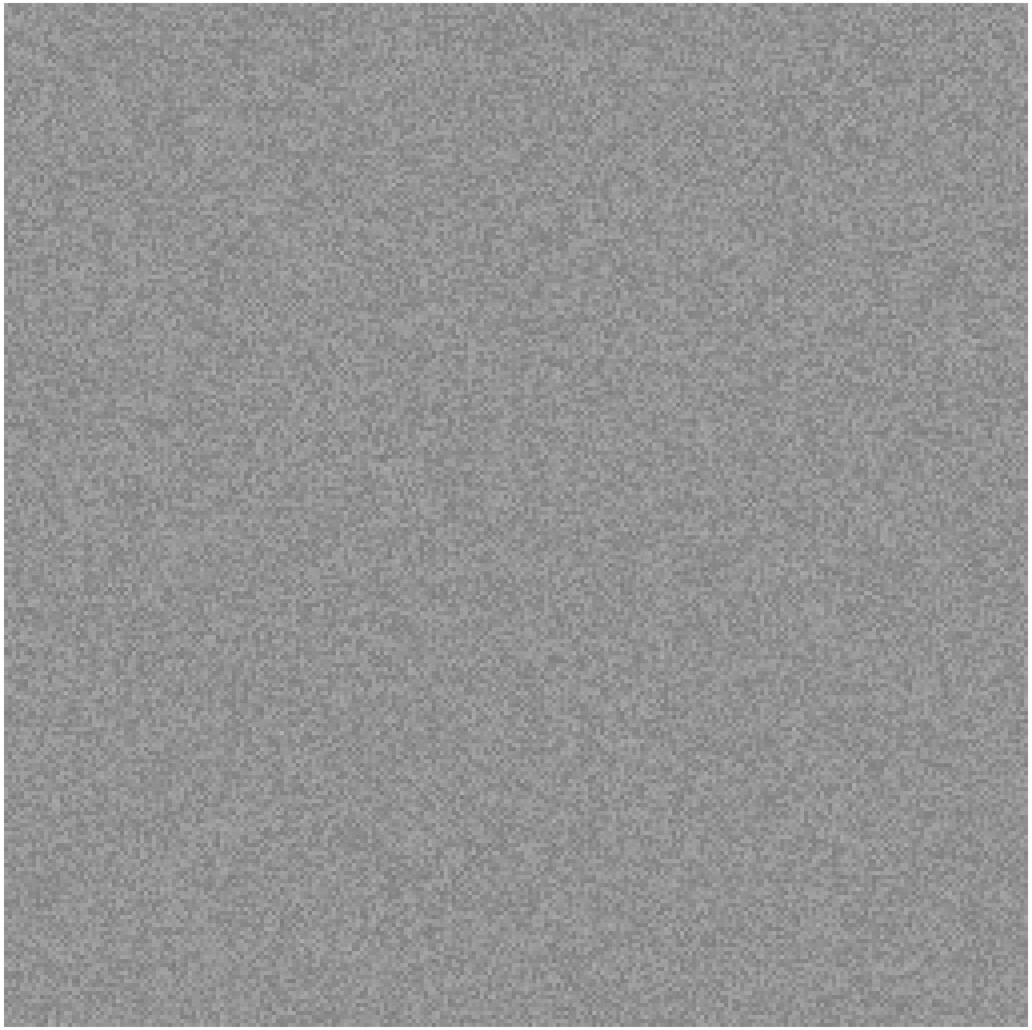


2次元では

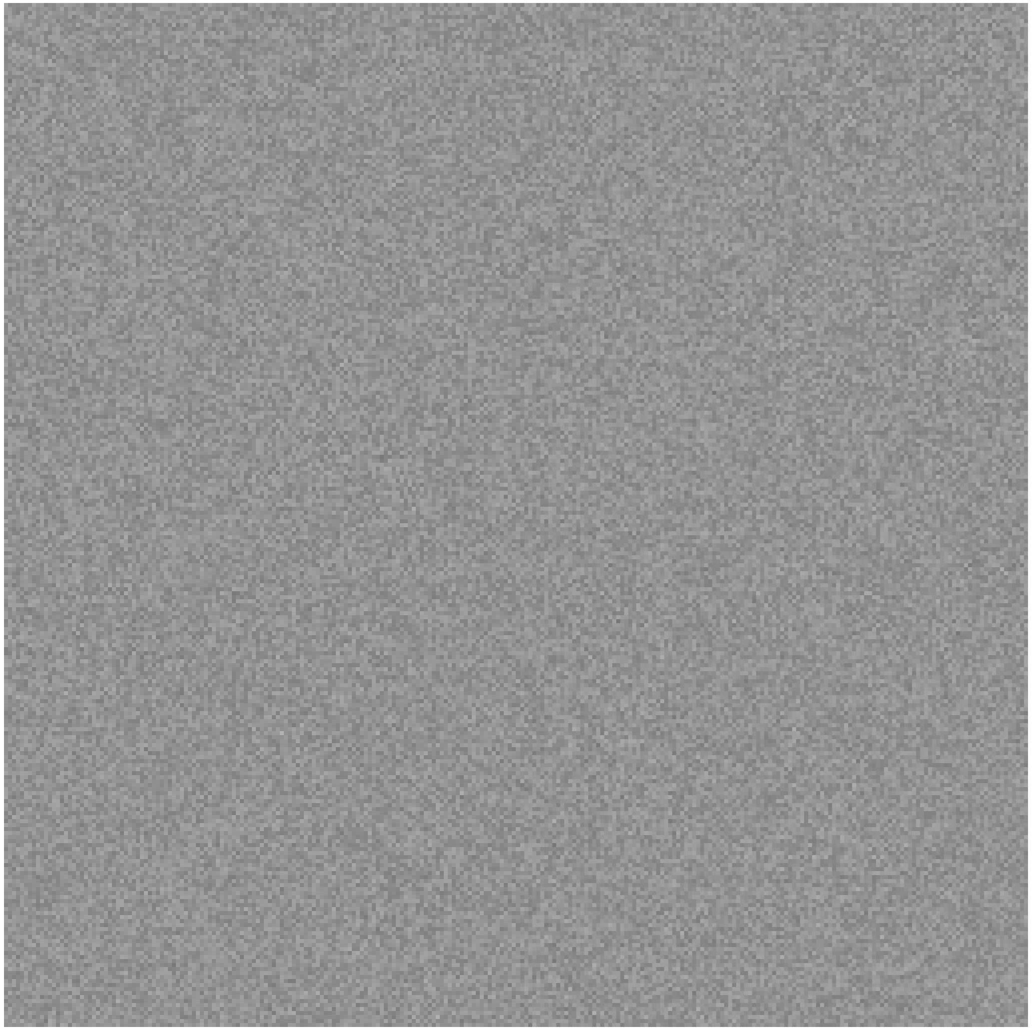
$a = 0$ $D_u = 1, D_v = 5$



$a = 0$ $D_u = 1, D_v = 20$



$a = 0$ $D_u = 2, D_v = 20$



$a = 0.05$ $D_u = 1, D_v = 20$