

分岐理論

おもな分岐の種類

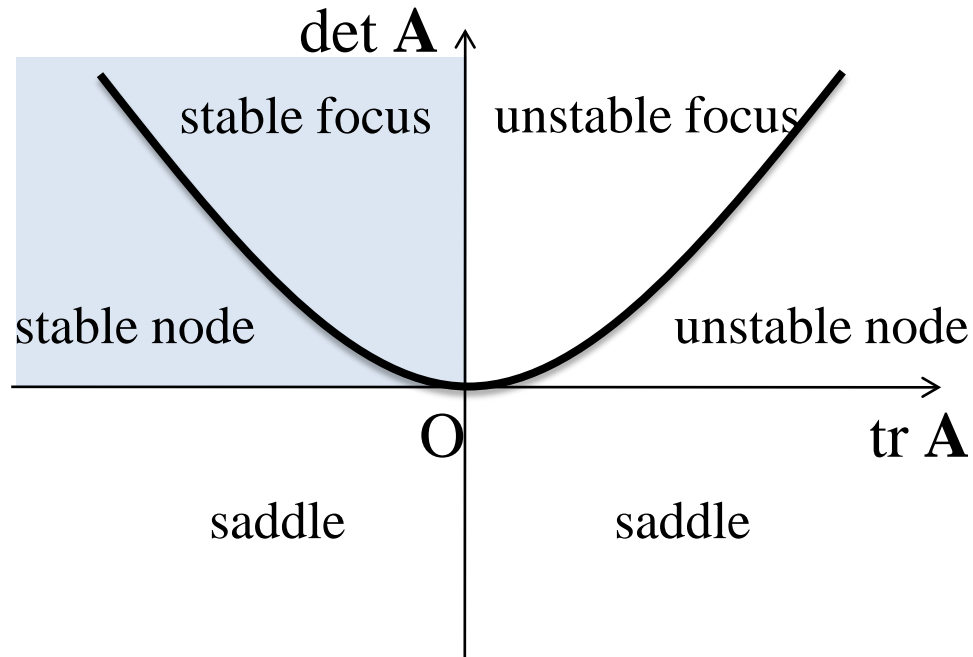
- サドル・ノード分岐
- ピッチフォーク分岐
- 安定性交替分岐(トランスクリティカル分岐)
- ホップ分岐

分岐とは?

2変数の力学系で考える

$$\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix}$$

\mathbf{A} : 関数行列



ただし、ホップ分岐以外では、 x 方向と y 方向は独立しているので、 $dy/dt = a y$ として考えることにする。

- サドル・ノード分岐

$$u = \pm \sqrt{\alpha}, v = 0 \quad (\alpha \geq 0) \text{ まわりで}$$

$$\frac{du}{dt} = \alpha - u^2$$

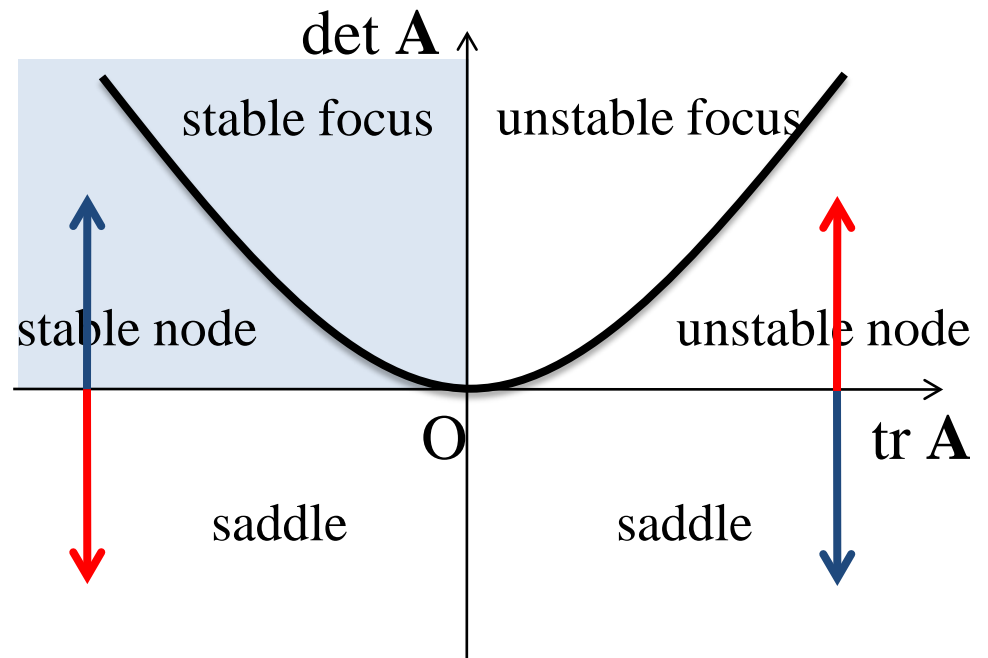
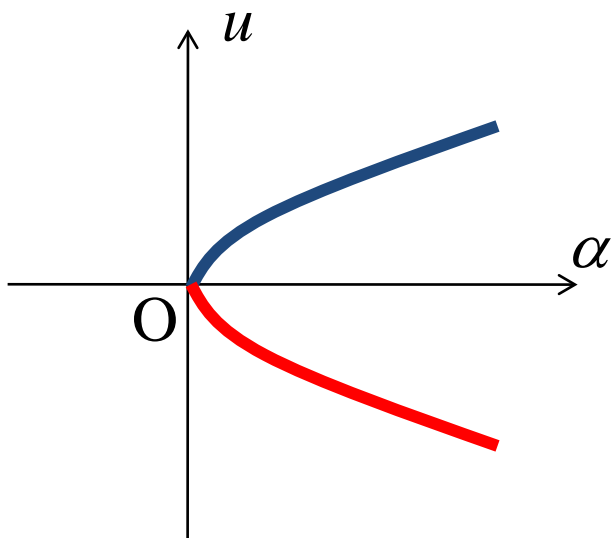
$$\frac{dv}{dt} = kv$$

固定点：

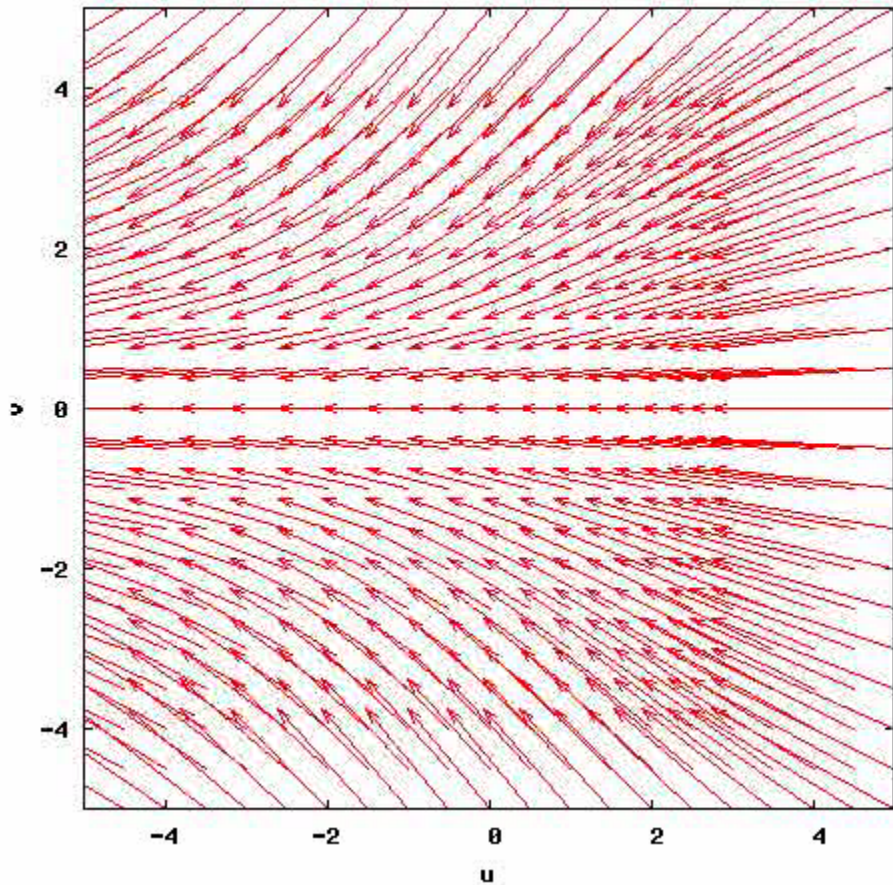
$$u = \pm \sqrt{\alpha}$$

$$v = 0$$

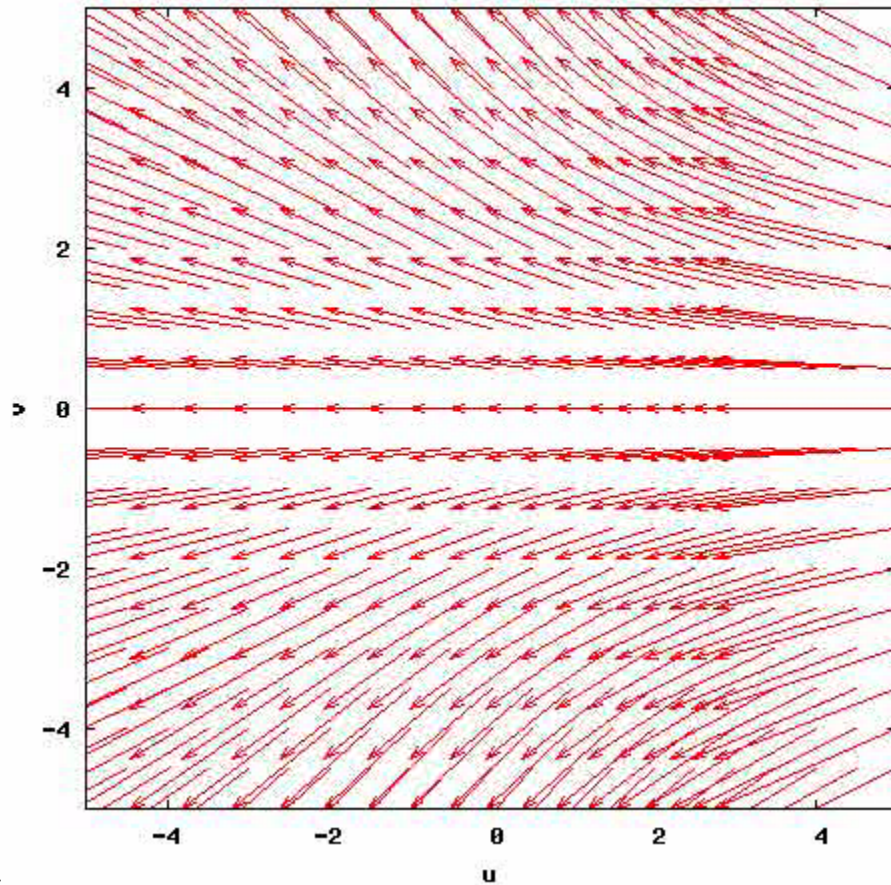
$$\mathbf{A} = \begin{pmatrix} \mp 2\sqrt{\alpha} & 0 \\ 0 & k \end{pmatrix}$$



alpha = -20.00



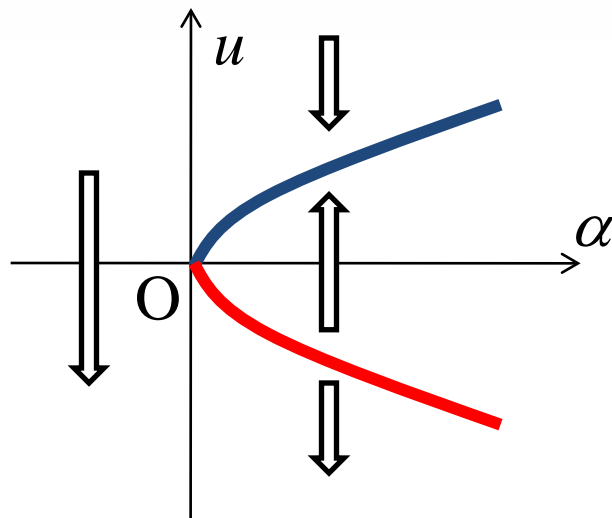
alpha = -20.00



$$\frac{du}{dt} = \alpha - u^2$$

$$\frac{dv}{dt} = kv$$

$k = -5$



$k = 5$

-ピッチフォーク分岐

$$u = \pm \sqrt{\alpha}, v = 0 \quad (\alpha \geq 0) \text{ まわりで}$$

$$\frac{du}{dt} = \alpha u - u^3$$

$$\frac{dv}{dt} = kv$$

固定点:

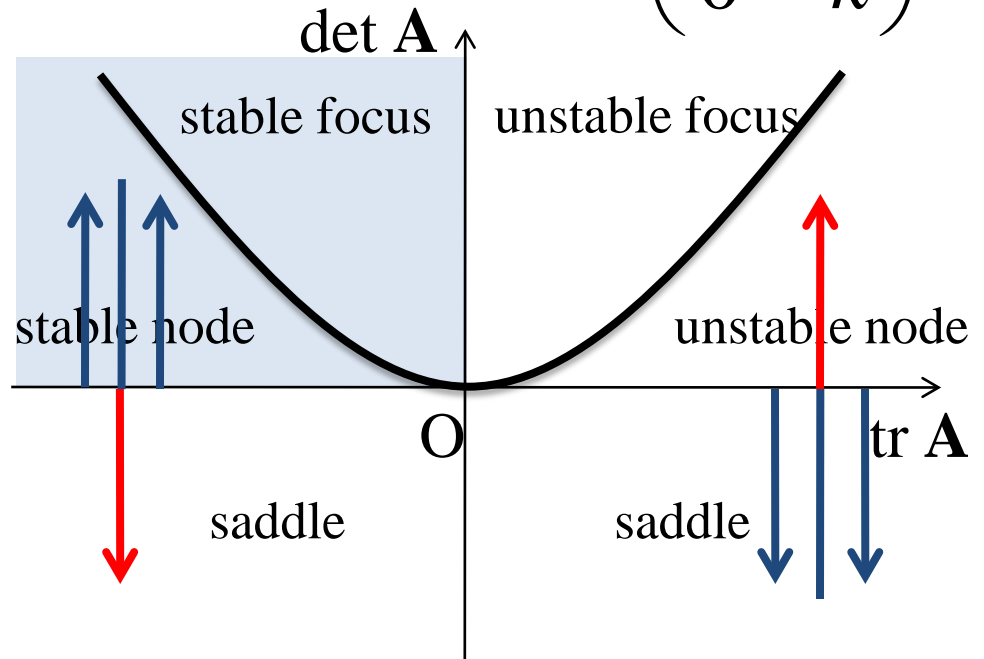
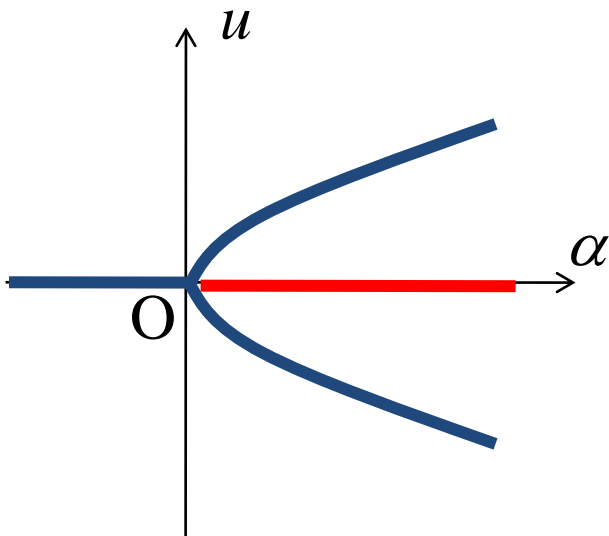
$$u = 0, \pm \sqrt{\alpha}$$

$$v = 0$$

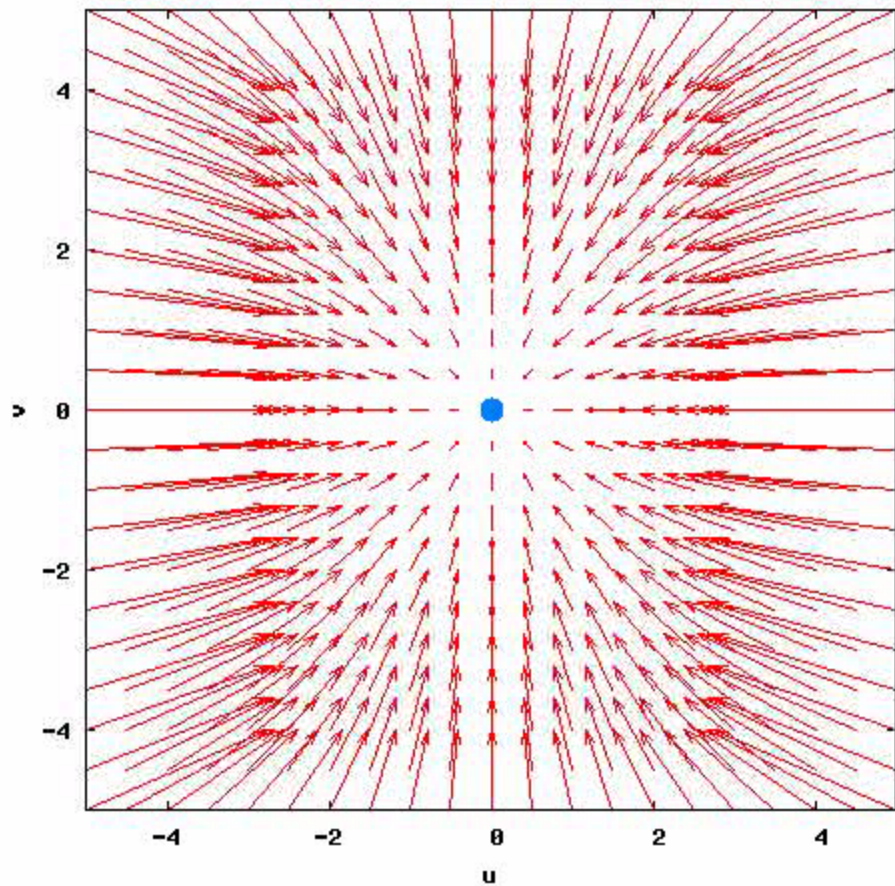
$$u = 0, v = 0 \text{ まわりで}$$

$$\mathbf{A} = \begin{pmatrix} -2\alpha & 0 \\ 0 & k \end{pmatrix}$$

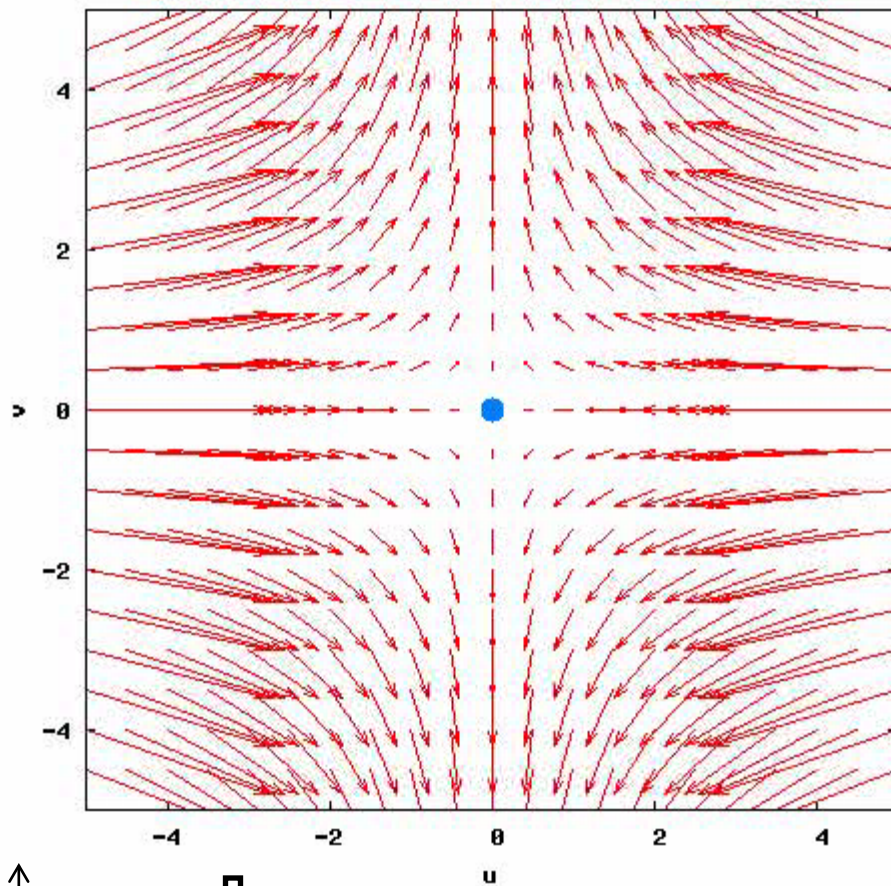
$$\mathbf{A} = \begin{pmatrix} \alpha & 0 \\ 0 & k \end{pmatrix}$$



alpha = -20.00

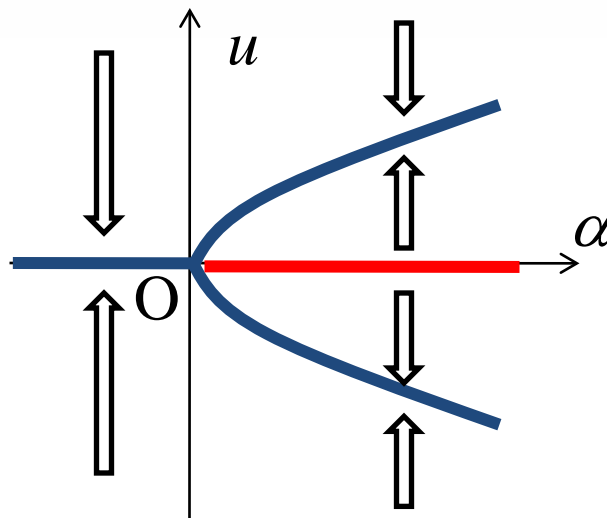


alpha = -20.00



$$\frac{du}{dt} = \alpha u - u^3 \quad k = -20$$

$$\frac{dv}{dt} = kv$$



$k = 20$

- ピッチフォーク分岐 (別タイプ)

$$u = \pm \sqrt{\alpha}, v = 0 \quad (\alpha \geq 0) \text{ まわりで}$$

$$\frac{du}{dt} = -\alpha u + u^3$$

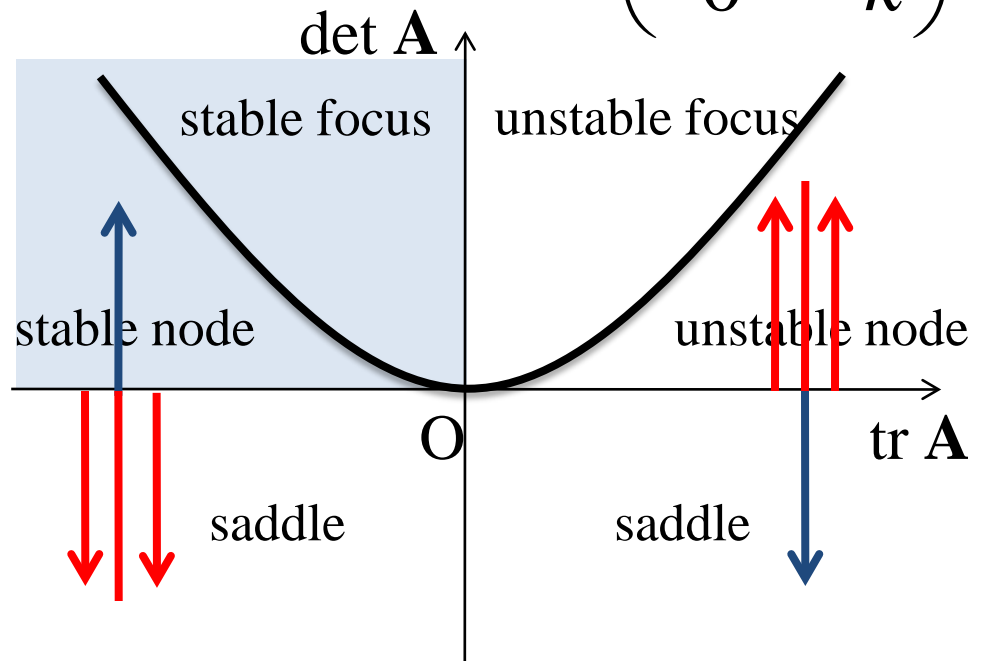
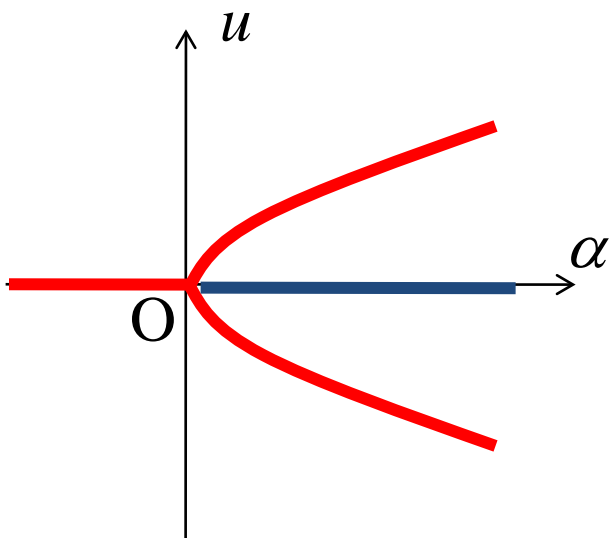
$$\mathbf{A} = \begin{pmatrix} 2\alpha & 0 \\ 0 & k \end{pmatrix}$$

$$\frac{dv}{dt} = kv$$

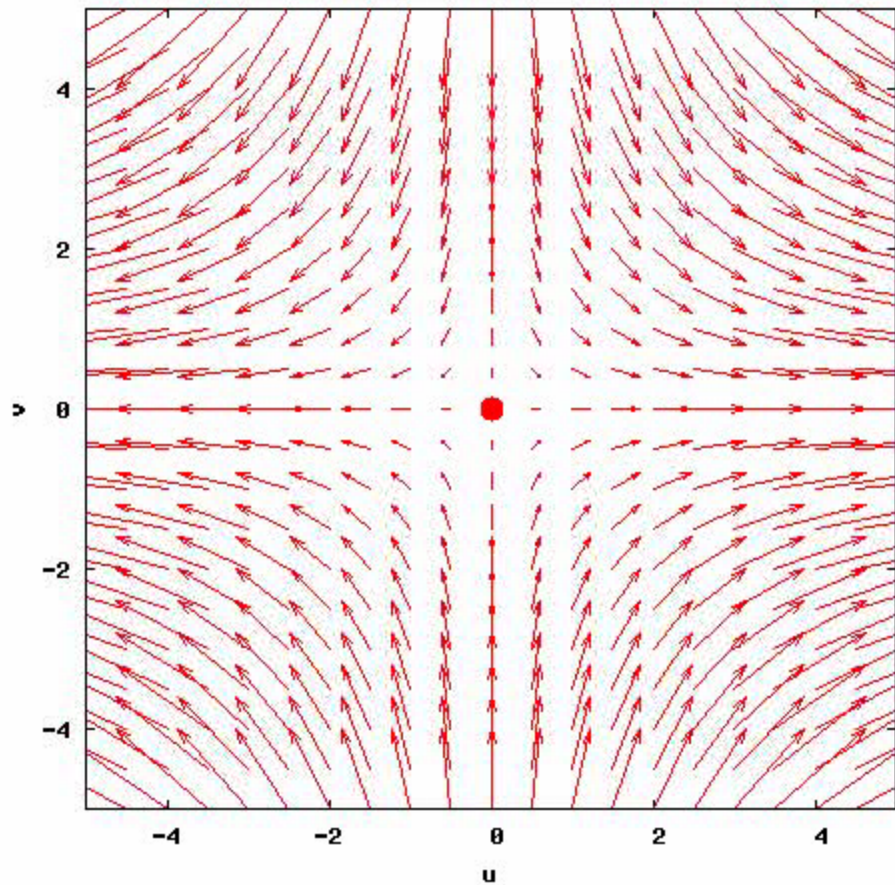
固定点 : $u = 0, \pm \sqrt{\alpha} \quad u = 0, v = 0$ まわりで

$$v = 0$$

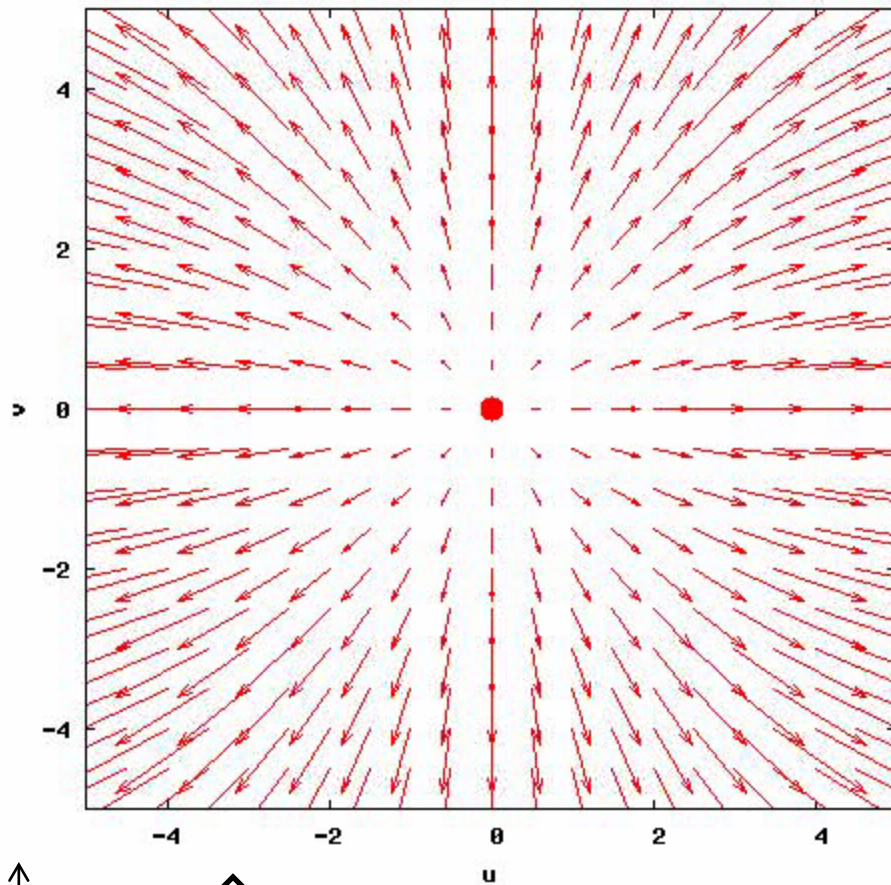
$$\mathbf{A} = \begin{pmatrix} -\alpha & 0 \\ 0 & k \end{pmatrix}$$



alpha = -20.00

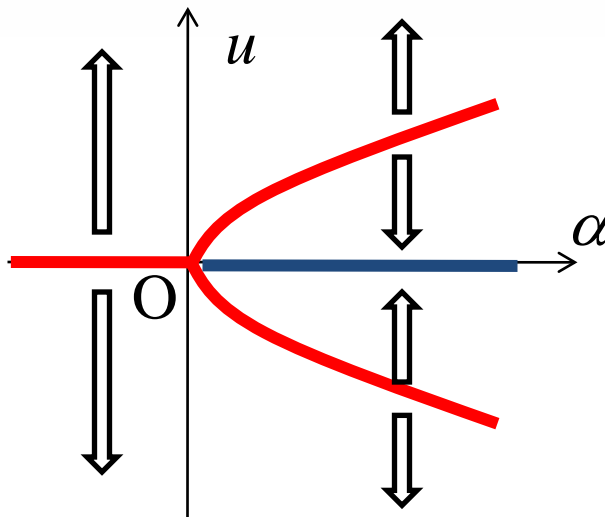


alpha = -20.00



$$\frac{du}{dt} = -\alpha u + u^3 \quad k = -20$$

$$\frac{dv}{dt} = kv$$



$k = 20$

- 安定性交替分岐

$$\frac{du}{dt} = \alpha u - u^2$$

$$\frac{dv}{dt} = kv$$

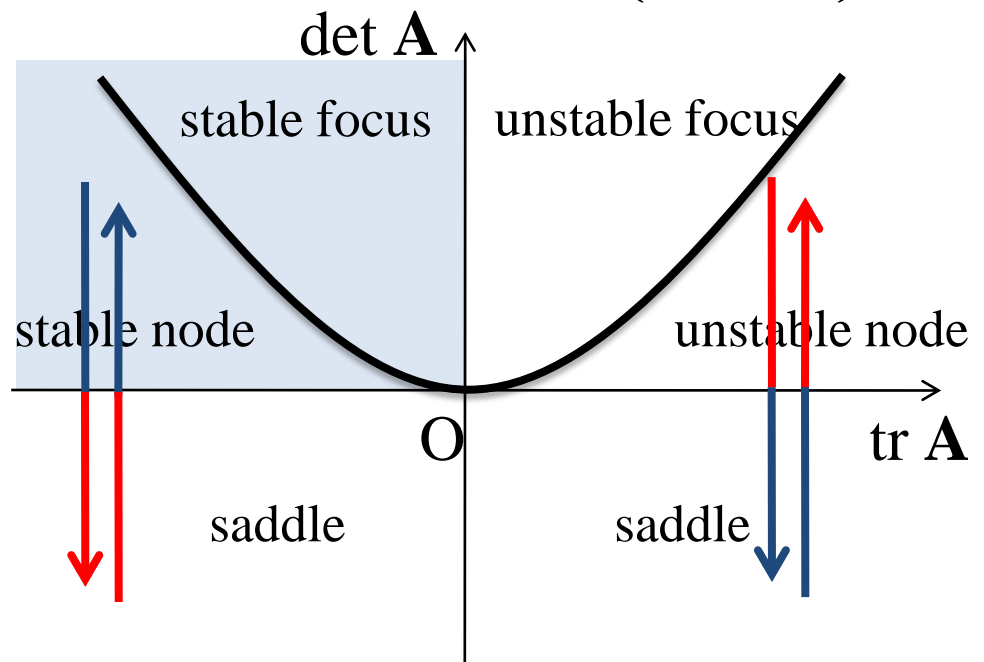
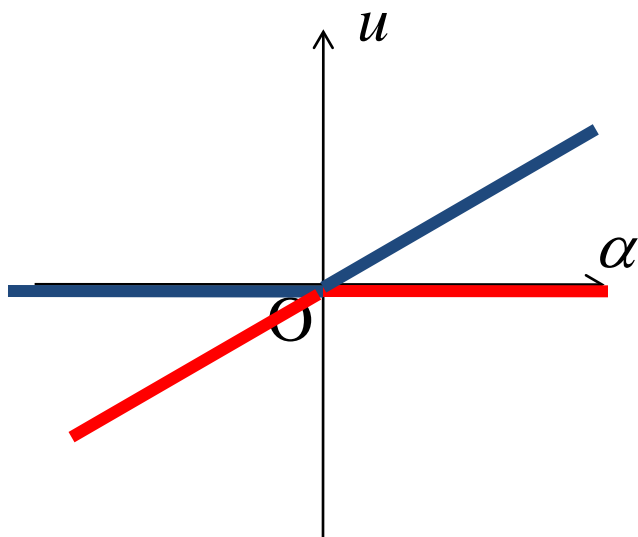
固定点：
 $u = 0, \lambda$
 $v = 0$

$u = \alpha, v = 0$ まわりで

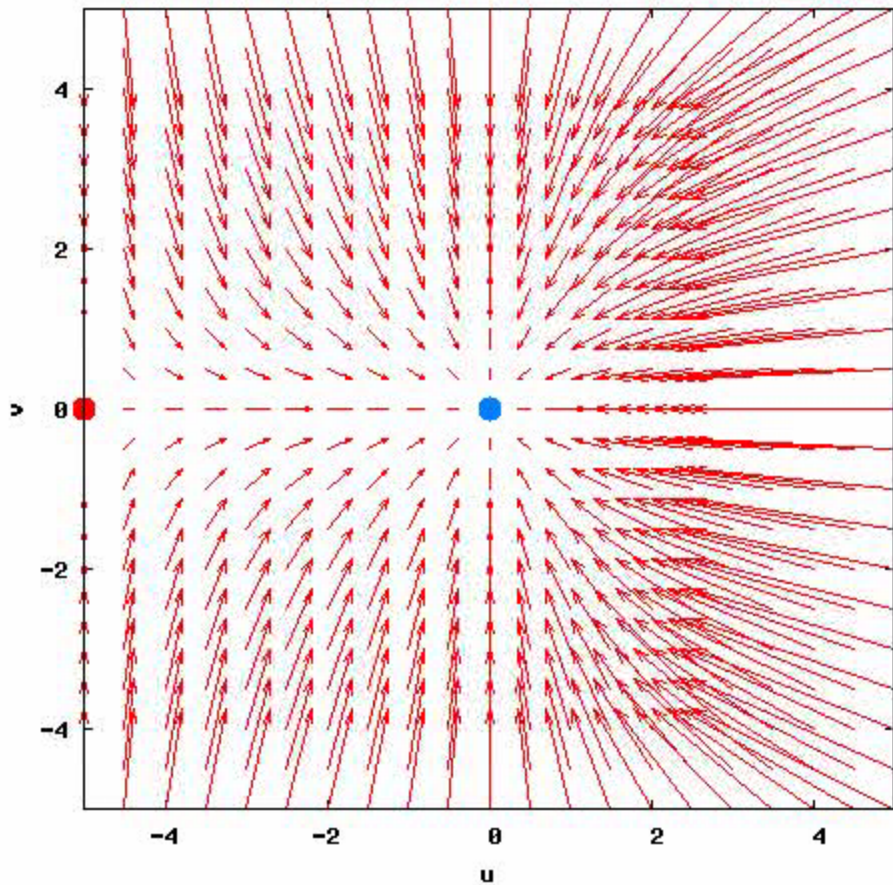
$$\mathbf{A} = \begin{pmatrix} -\alpha & 0 \\ 0 & k \end{pmatrix}$$

$u = 0, v = 0$ まわりで

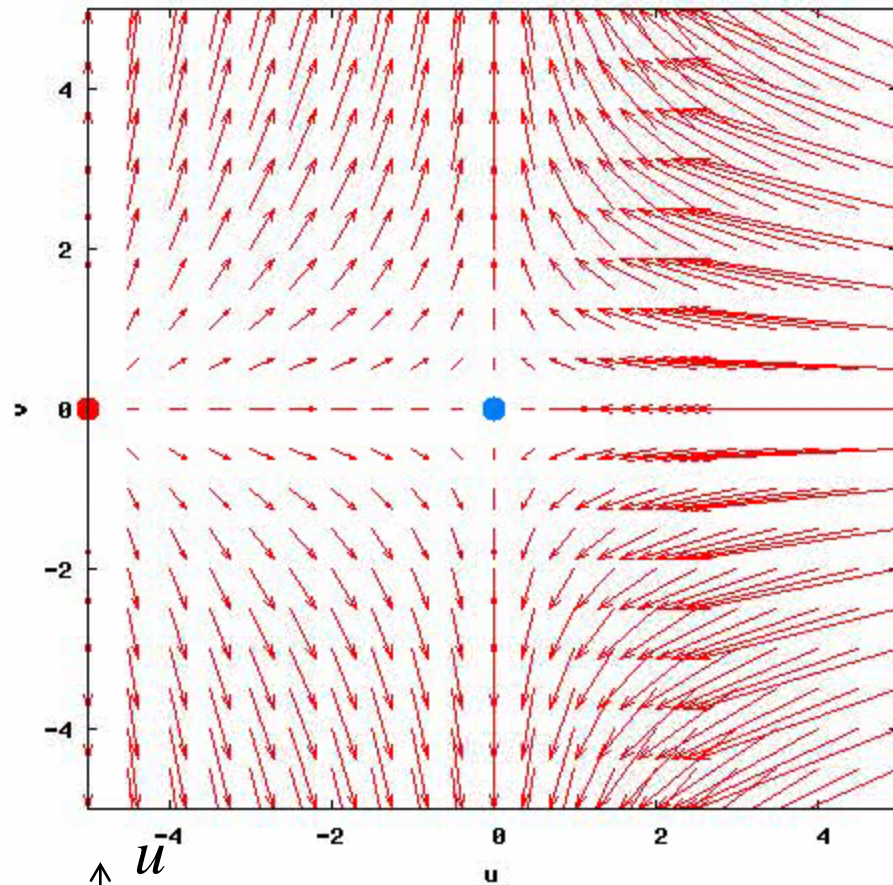
$$\mathbf{A} = \begin{pmatrix} \alpha & 0 \\ 0 & k \end{pmatrix}$$



alpha = -5.00

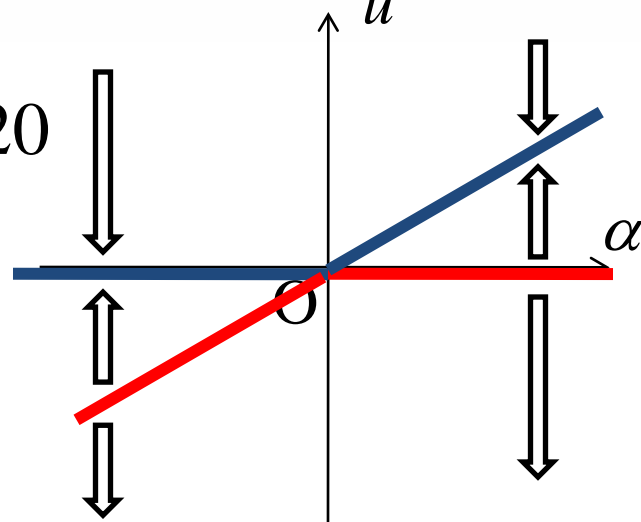


alpha = -5.00



$$\frac{du}{dt} = -\alpha u + u^3 \quad k = -20$$

$$\frac{dv}{dt} = kv$$



$k = 20$

- ホップ分岐

固定点 :

$$u = 0 \quad v = 0$$

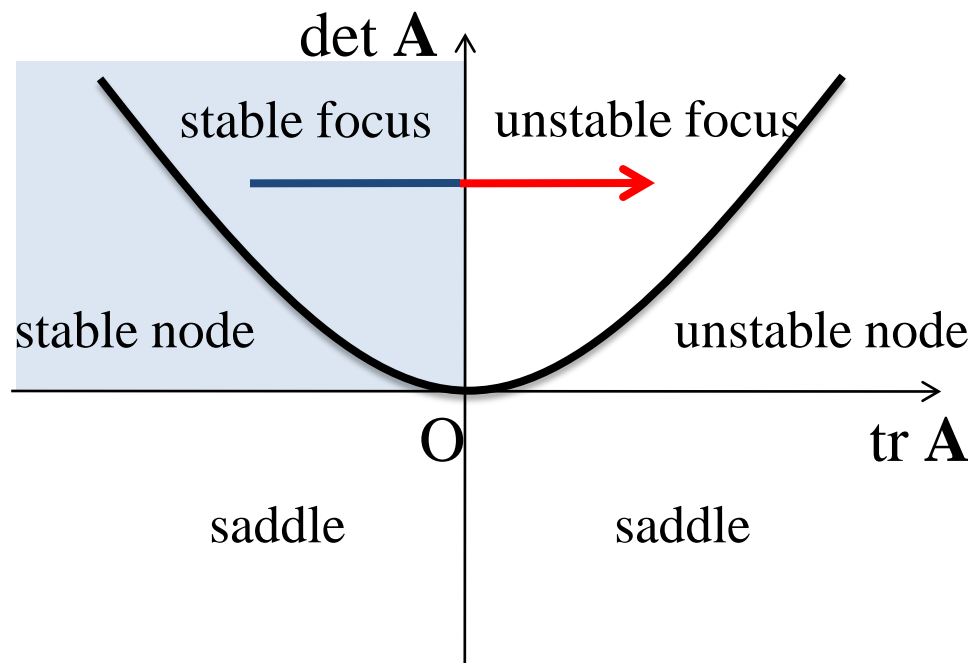
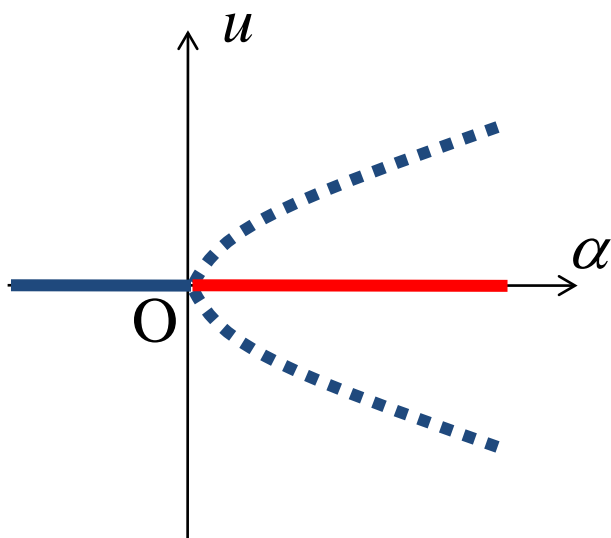
$u = 0, v = 0$ まわりで

$$\frac{du}{dt} = -v + \alpha u - (u^2 + v^2)(u - \beta v)$$

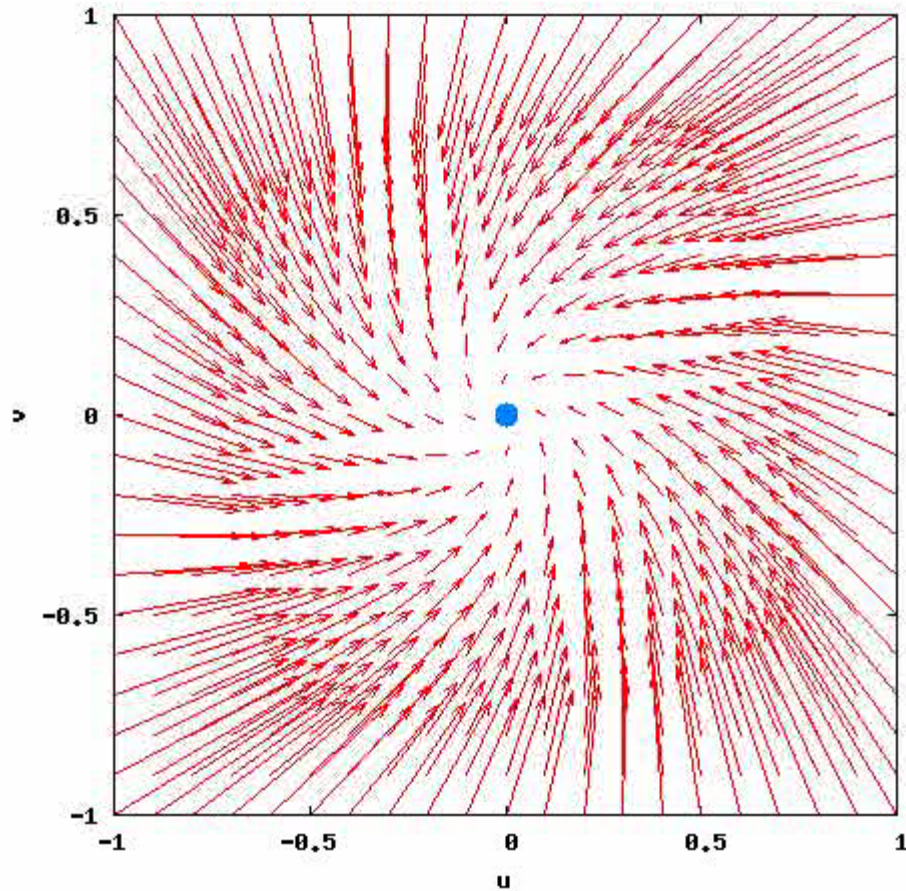
$$\frac{dv}{dt} = u + \alpha v - (u^2 + v^2)(v + \beta u)$$

$$\mathbf{A} = \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix}$$

$$\lambda = \alpha \pm i$$

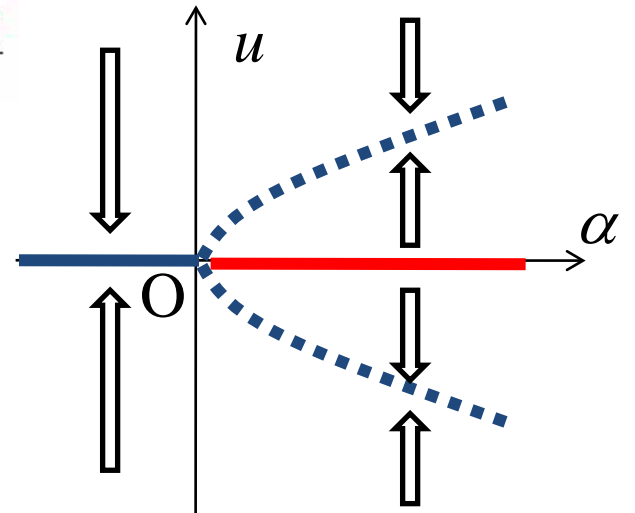


alpha = -2.00



$\beta = 0$

$$\frac{du}{dt} = -v + \alpha u - (u^2 + v^2)(u - \beta v)$$
$$\frac{dv}{dt} = u + \alpha v - (u^2 + v^2)(v + \beta u)$$



- ホップ分岐 (別タイプ)

固定点 :

$$u = 0 \quad v = 0$$

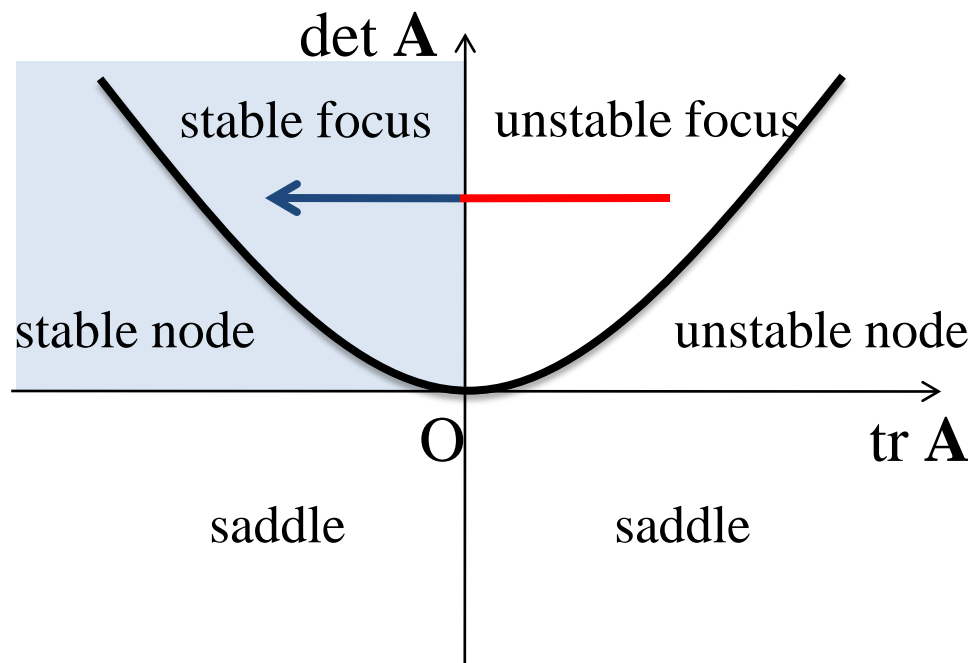
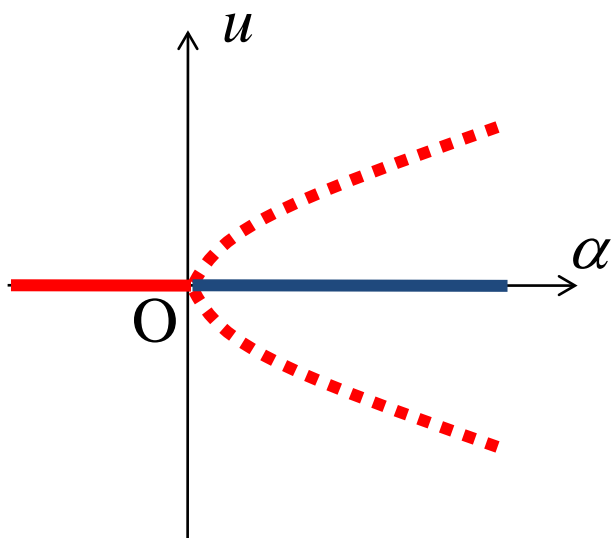
$$\frac{du}{dt} = -v - \alpha u + (u^2 + v^2)(u - \beta v)$$

$$\frac{dv}{dt} = u - \alpha v + (u^2 + v^2)(v + \beta u)$$

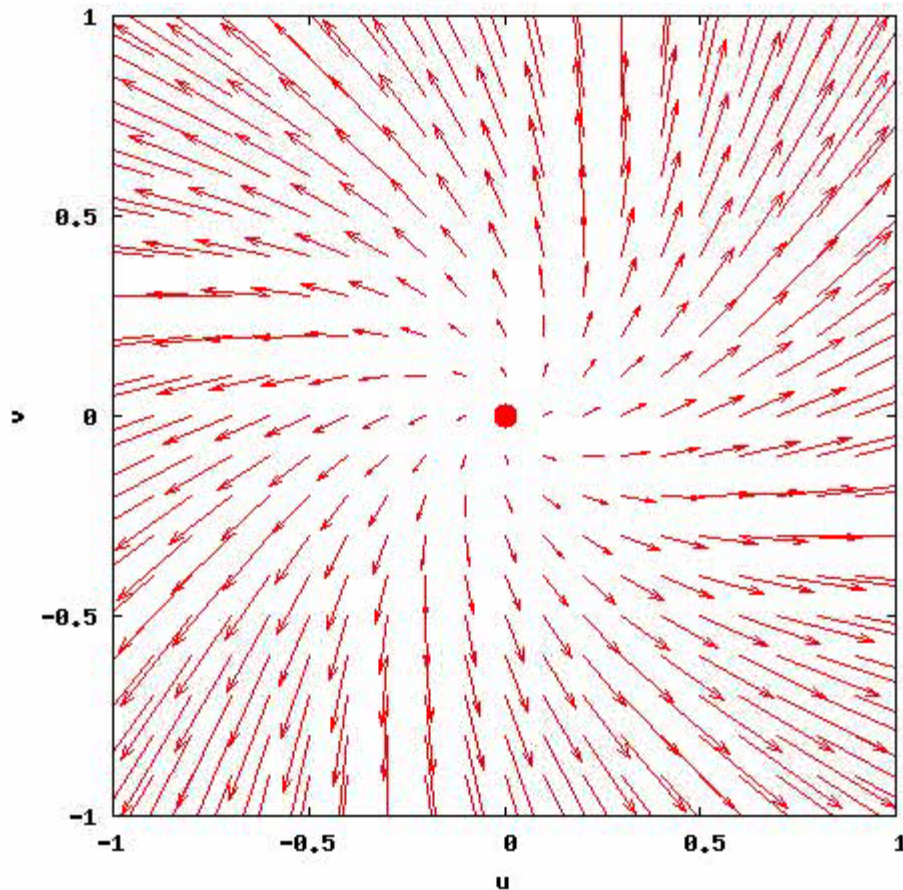
$u = 0, v = 0$ まわりで

$$\mathbf{A} = \begin{pmatrix} -\alpha & -1 \\ 1 & -\alpha \end{pmatrix}$$

$$\lambda = -\alpha \pm i$$

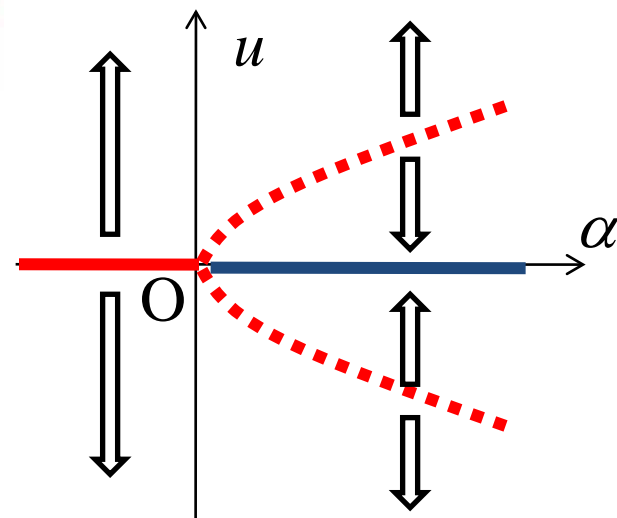


alpha = -2.00



$\beta = 0$

$$\frac{du}{dt} = -v - \alpha u + (u^2 + v^2)(u - \beta v)$$
$$\frac{dv}{dt} = u - \alpha v + (u^2 + v^2)(v + \beta u)$$



別の面からみた分岐の特徴付け

- スーパークリティカル(超臨界)分岐
- サブクリティカル(亜臨界)分岐