

2009.1.12
物性物理学C

反応拡散系とパターン形成

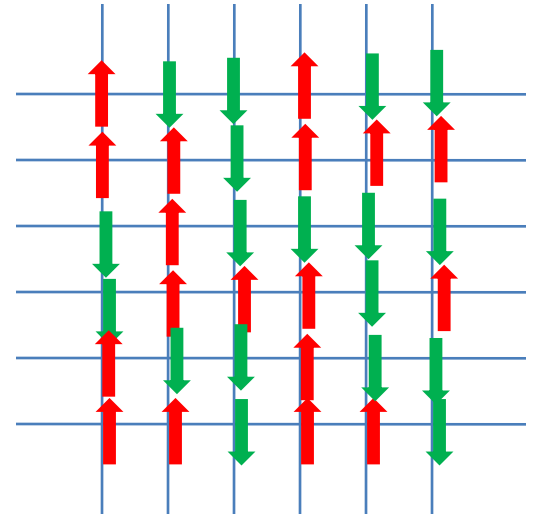
スピン系での局所ポテンシャル

上向きスピンの割合: ϕ

サイト数: N

最近接サイト数: a

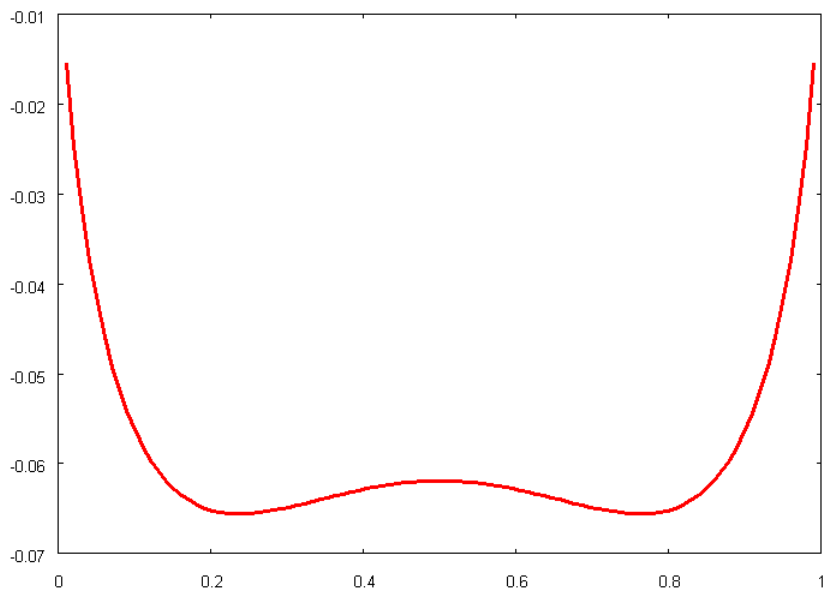
相互作用エネルギーの差: ε



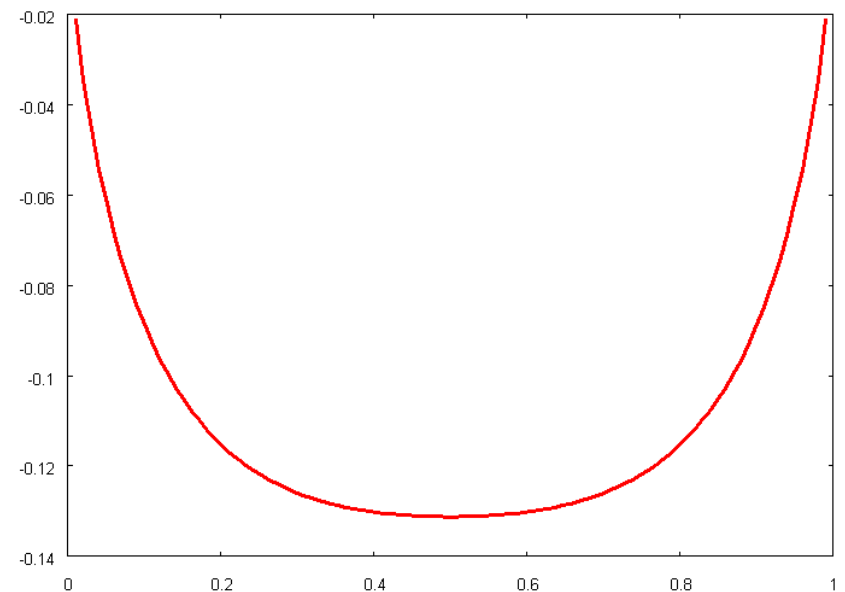
$$F = Na\varepsilon\phi(1-\phi) - Nk_B T[\phi \ln \phi + (1-\phi)\ln(1-\phi)]$$

ポテンシャルの形状

$T < T_c$

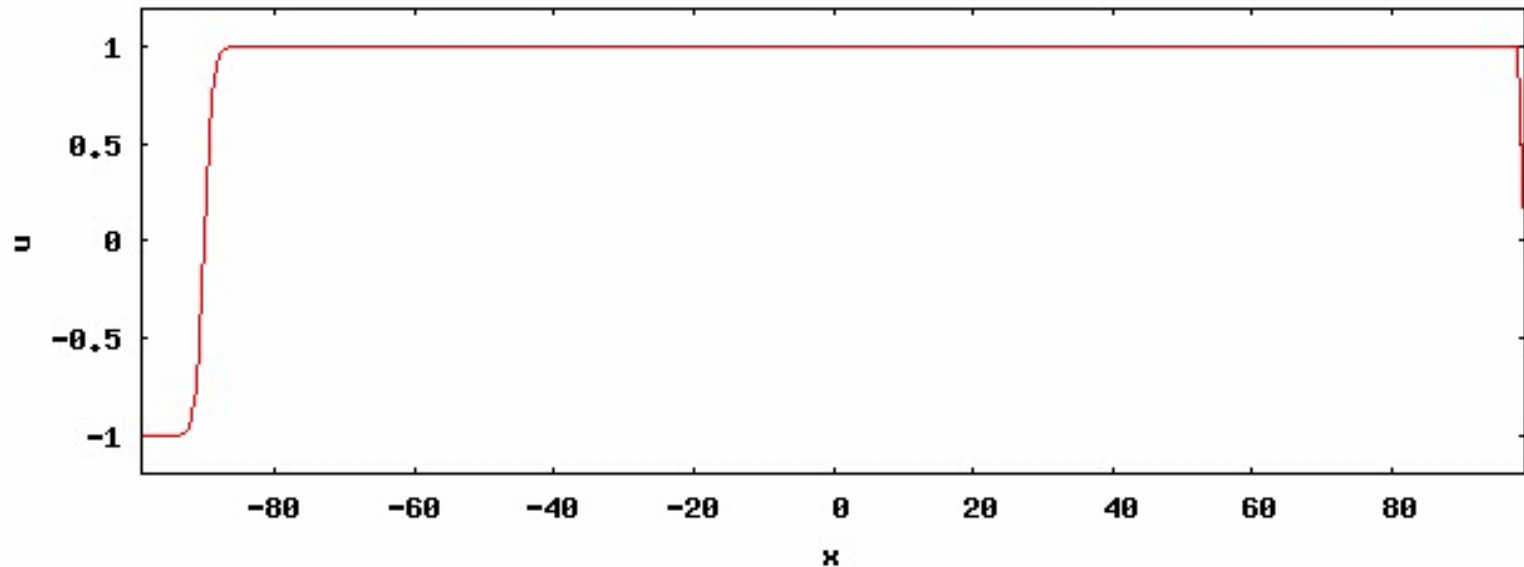


$T > T_c$

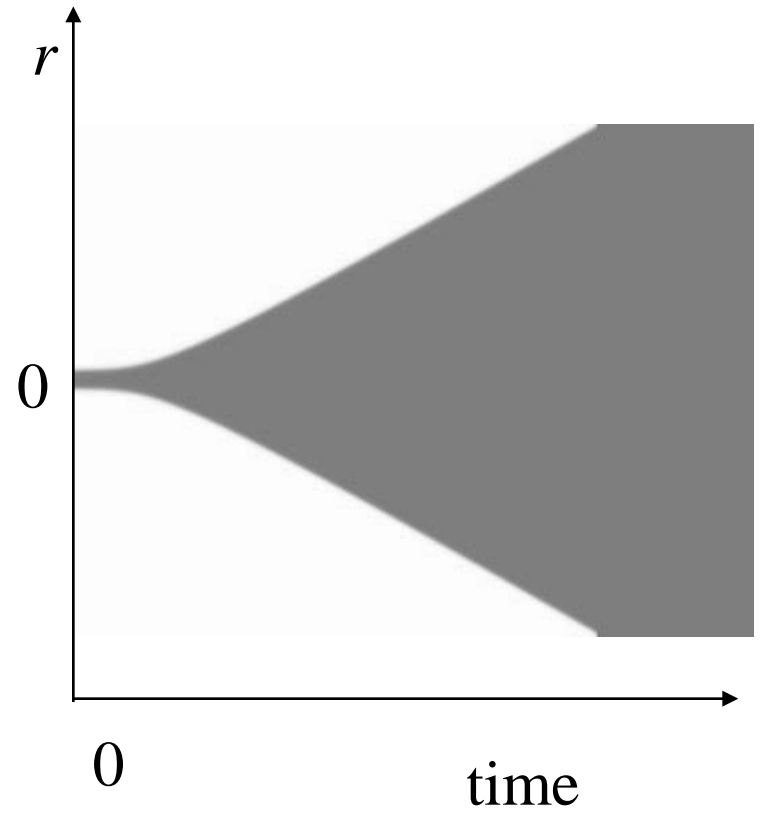


1次元系での伝播

$$\frac{\partial u}{\partial t} = K \left[u - u^3 - P + \varepsilon \nabla^2 u \right]$$



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Turingパターン(静止パターン)

$$\frac{\partial u}{\partial t} = -u^3 + u - 4v + D_u \nabla^2 u$$

$$\frac{\partial v}{\partial t} = u - 3v - a + D_v \nabla^2 v$$

$a = 0$ では $u = v = 0$ が固定点

$$\frac{d}{dt} \begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix}$$

$\text{tr } A = -2$ 、 $\text{det } A = 1$ より安定

$$u = 0 + \int dk \Delta u(k) e^{ikx}$$

$$v = 0 + \int dk \Delta v(k) e^{ikx} \quad \text{と} \text{お} \text{い} \text{て}$$

$$\frac{d}{dt} \begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = \begin{pmatrix} 1 - D_u k^2 & -4 \\ 1 & -3 - D_u k^2 \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix}$$

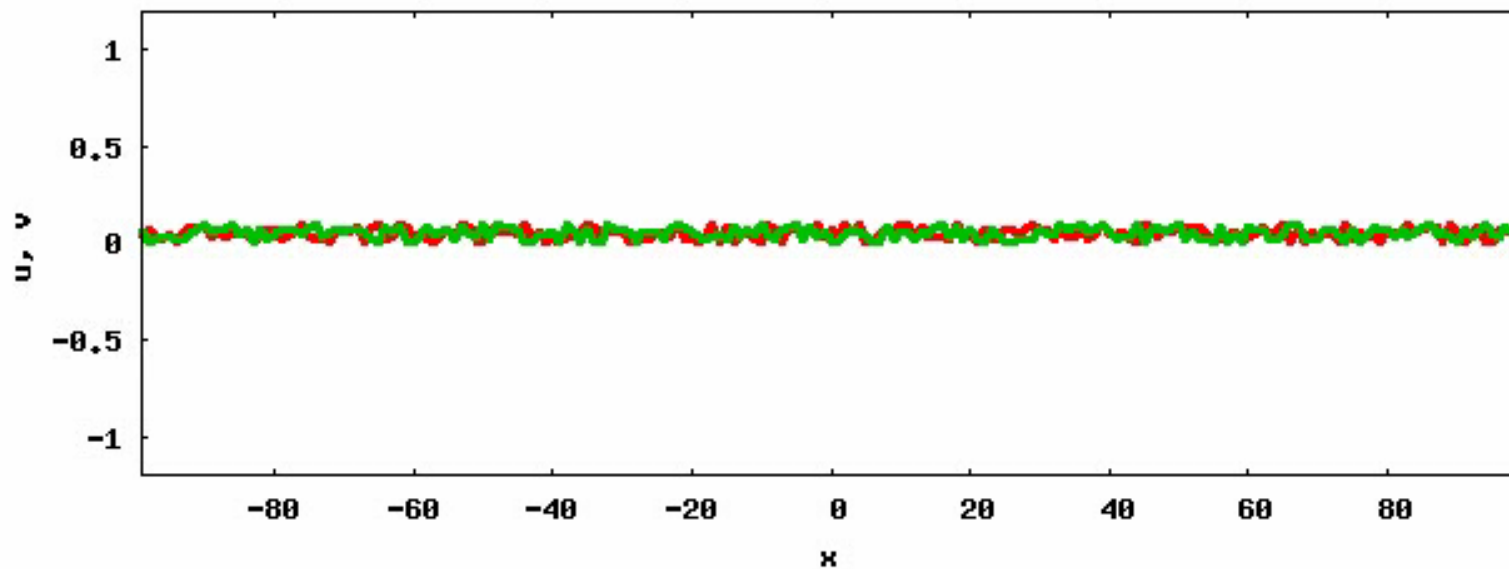
不安定化する条件は

$$D_v > 9D_u$$

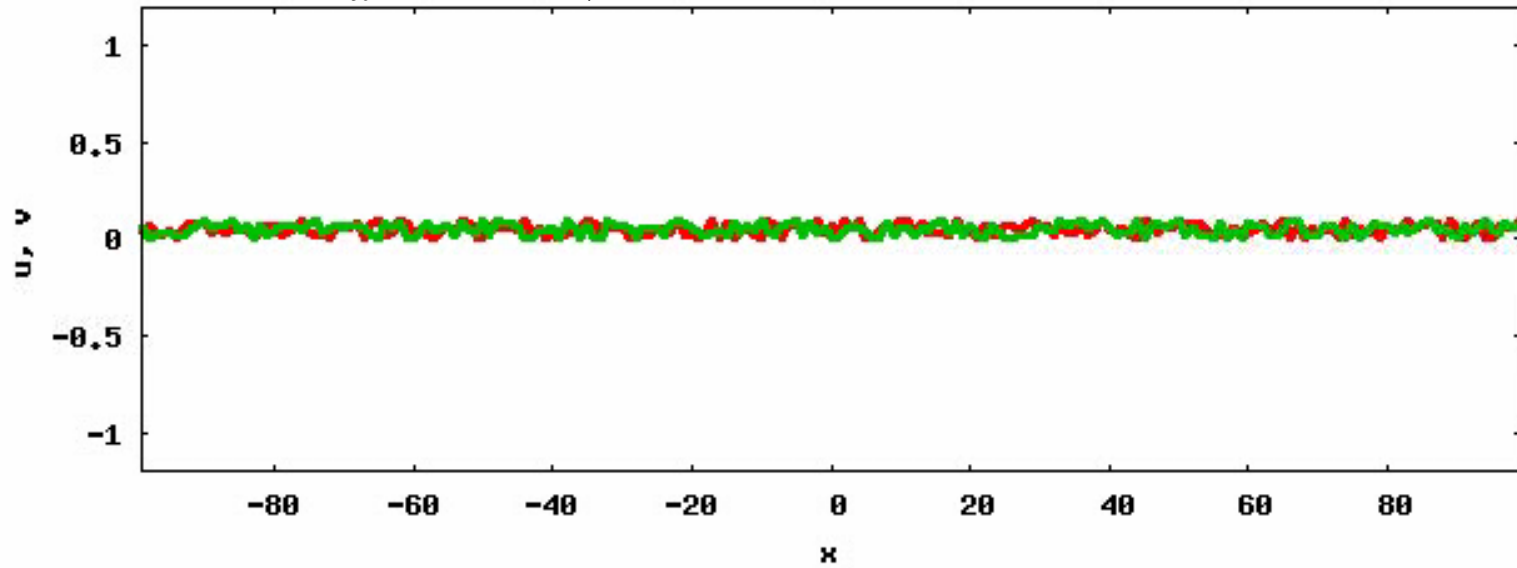
はじめに不安定化する波数は $k = \sqrt{\frac{1}{2D_u} - \frac{3}{2D_v}}$

1次元では

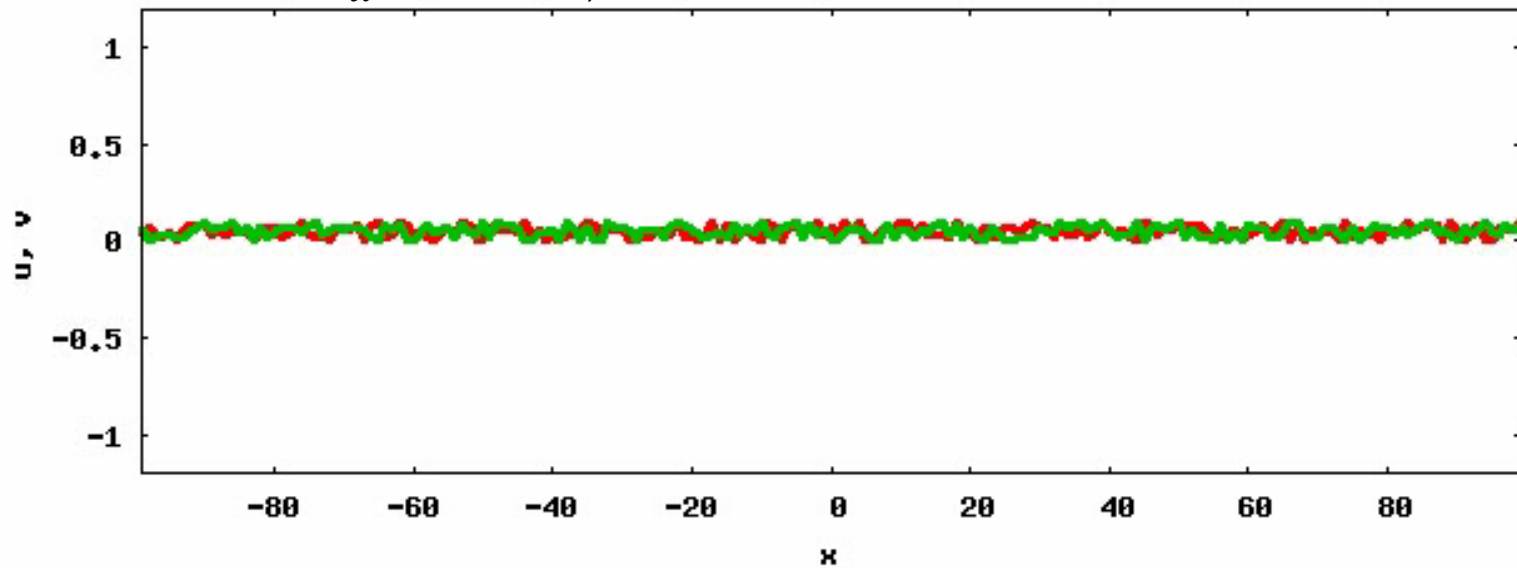
$$a = 0, \quad D_u = 1, D_v = 5$$



$$a = 0, \quad D_u = 1, D_v = 20$$



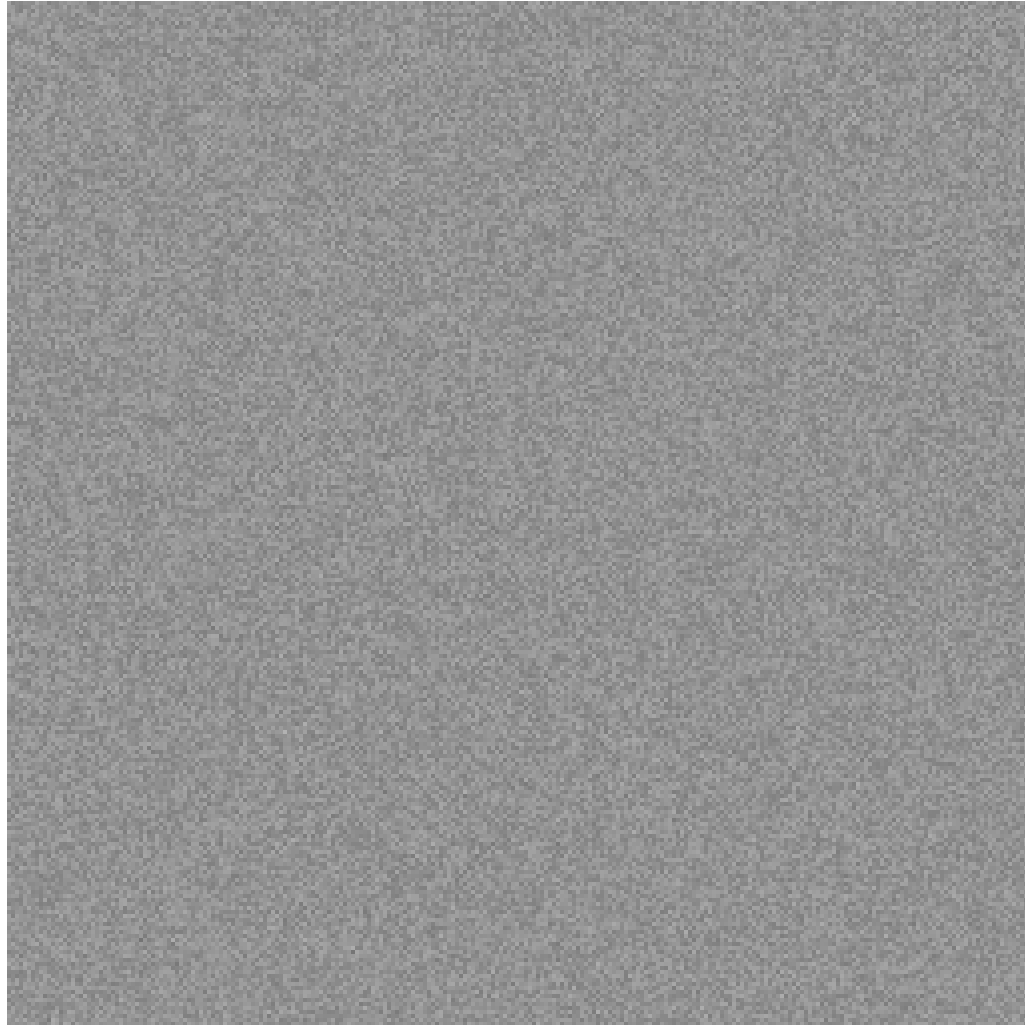
$$a = 0, \quad D_u = 2, D_v = 20$$



2次元では

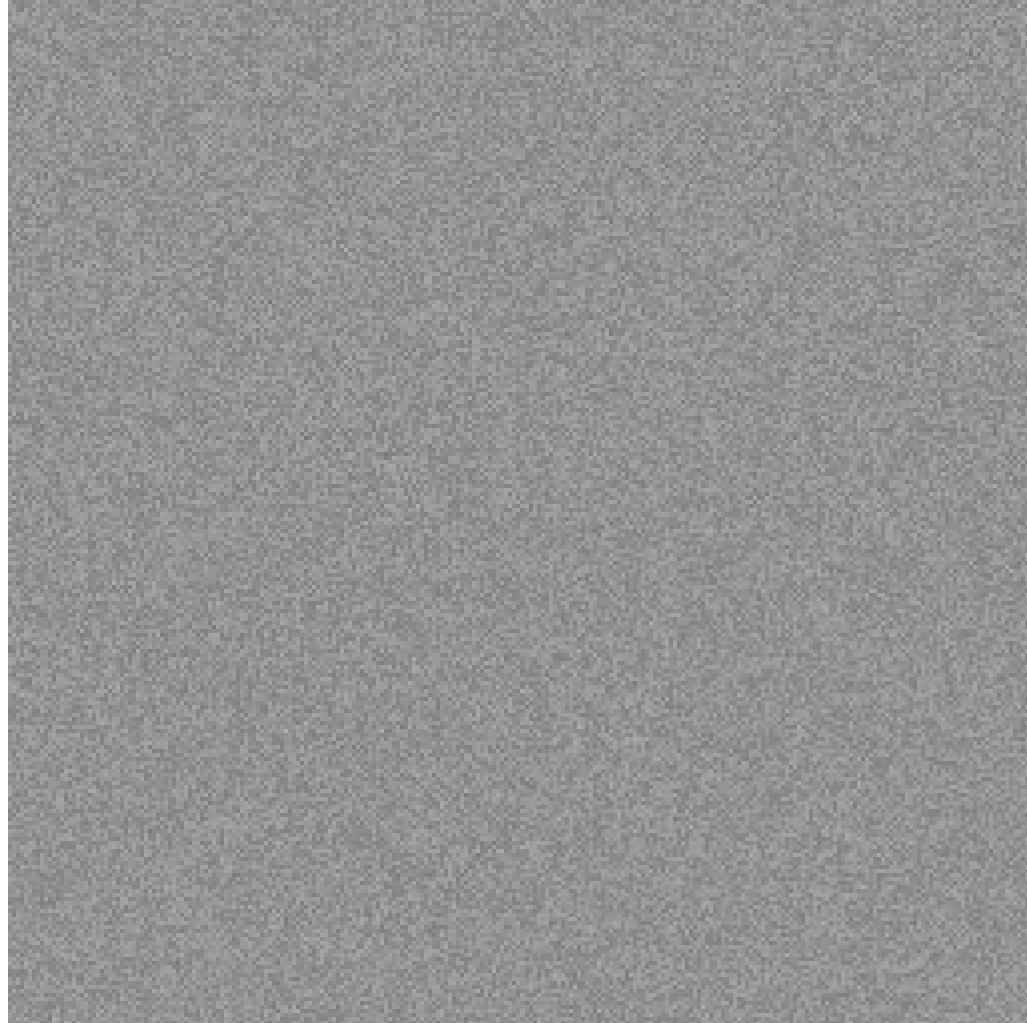
$$a = 0$$

$$D_u = 1, D_v = 5$$



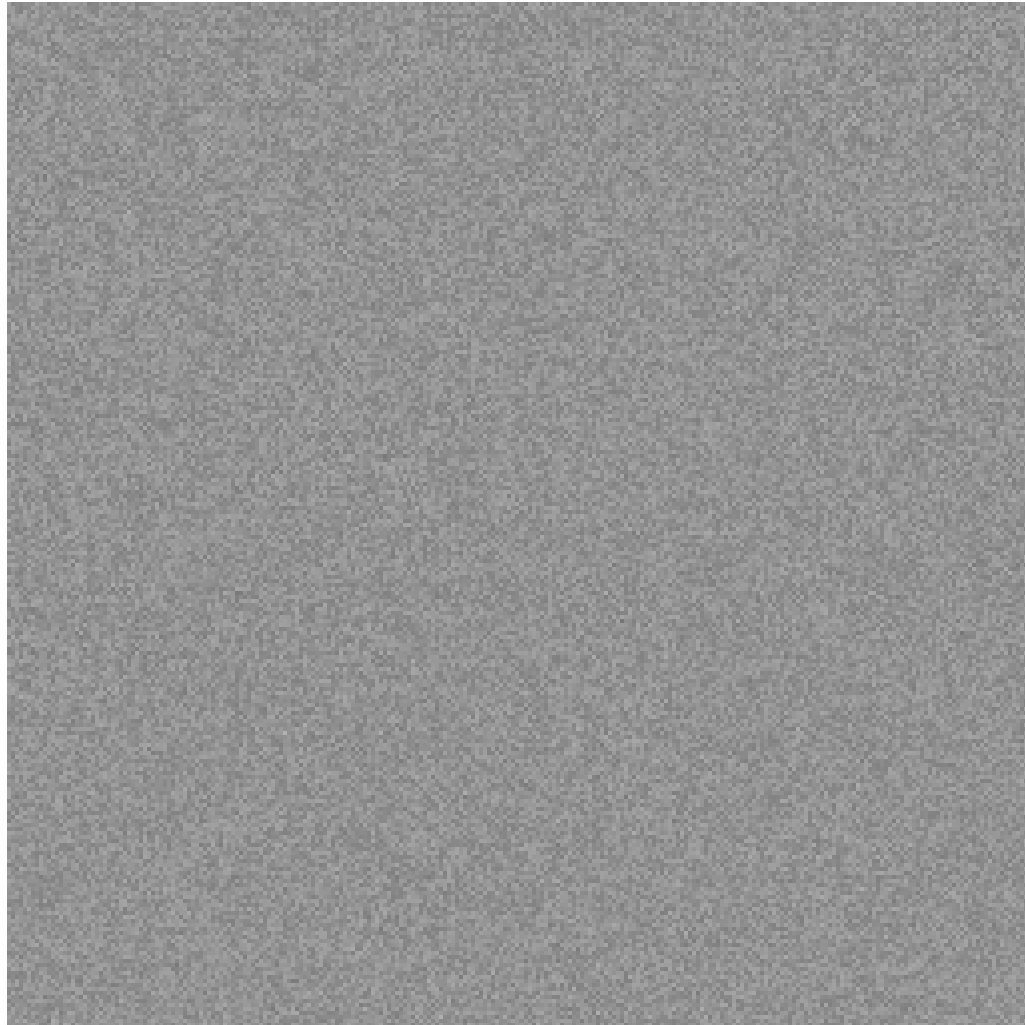
$$a = 0$$

$$D_u = 1, D_v = 20$$



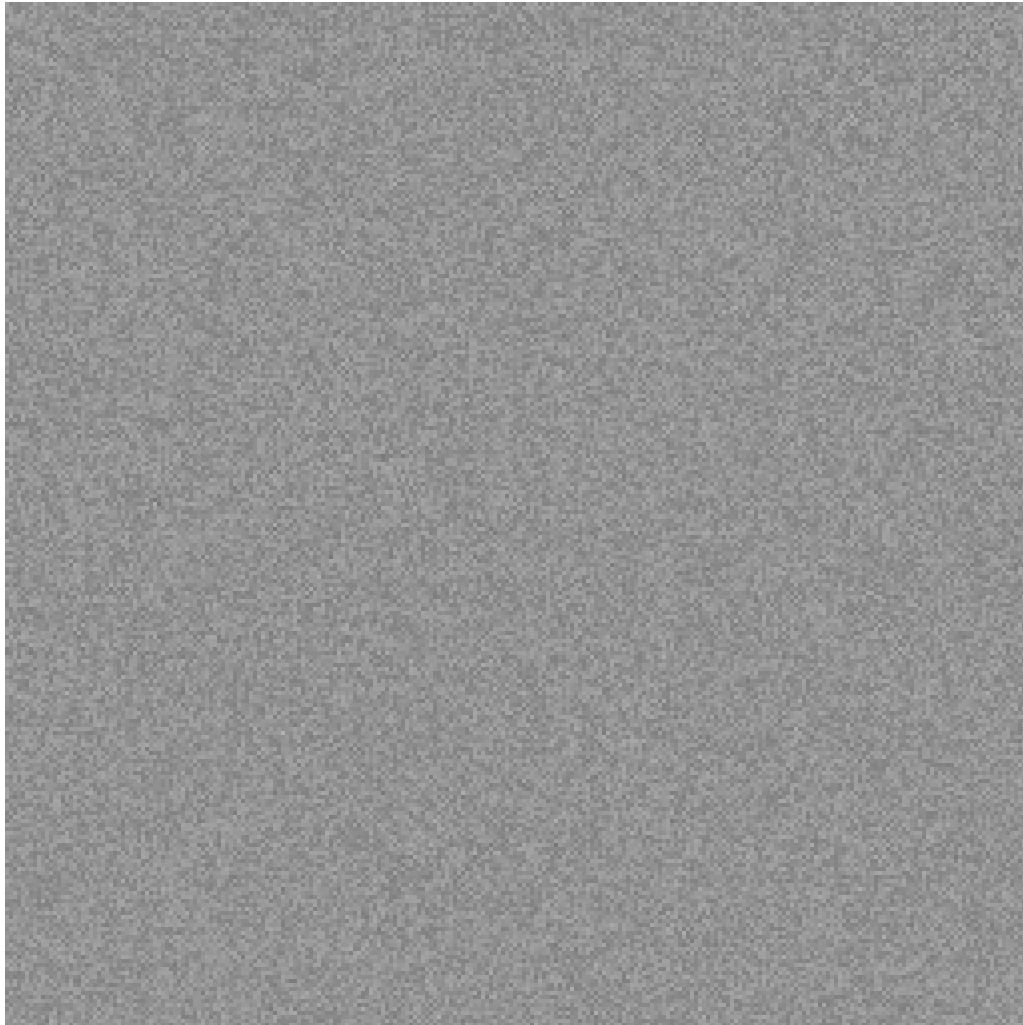
$$a = 0$$

$$D_u = 2, D_v = 20$$



$$a = 0.05$$

$$D_u = 1, D_v = 20$$



Belousov-Zhabotinsky反応

a) Target Pattern



(4倍速)

3 mm

b) Spiral Pattern



(4倍速)

3 mm

$$\frac{\partial U}{\partial t} = \frac{1}{\varepsilon} \left(U(1-U) - fV \frac{U-q}{U+q} \right) + D_U \nabla^2 U$$

$$\frac{\partial V}{\partial t} = U - V + D_V \nabla^2 V$$

Target Pattern



Spiral Pattern



(Keener-Tyson version Oregonatorを使用)