2012.12.11

分岐理論

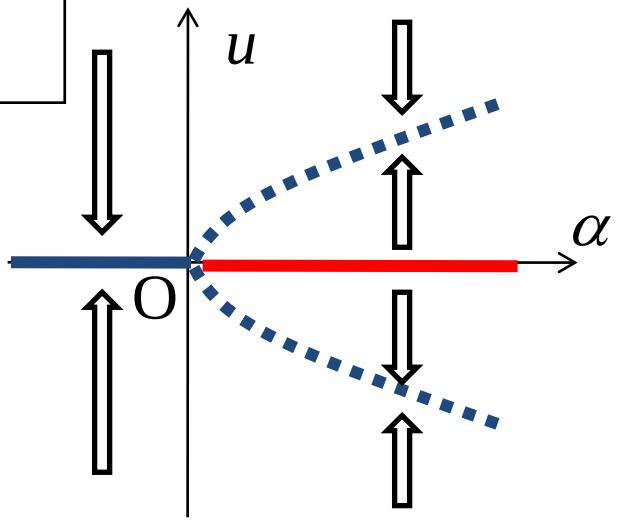
おもな分岐の種類

- サドル・ノード分岐
- ピッチフォーク分岐
- トランスクリティカル分岐（安定性交替分岐 ）
- ホップ分岐

$$\beta = 0$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -v + \alpha u - \left(u^2 + v^2\right)\left(u - \beta v\right)$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = u + \alpha v - \left(u^2 + v^2\right)\left(v + \beta u\right)$$



非線形振動子と同期現象

Stuart-Landau方程式

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = ax - \omega y - \left(x^2 + y^2\right)\left(x - by\right) \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = ay + \omega x - \left(x^2 + y^2\right)\left(y + bx\right) \end{cases}$$

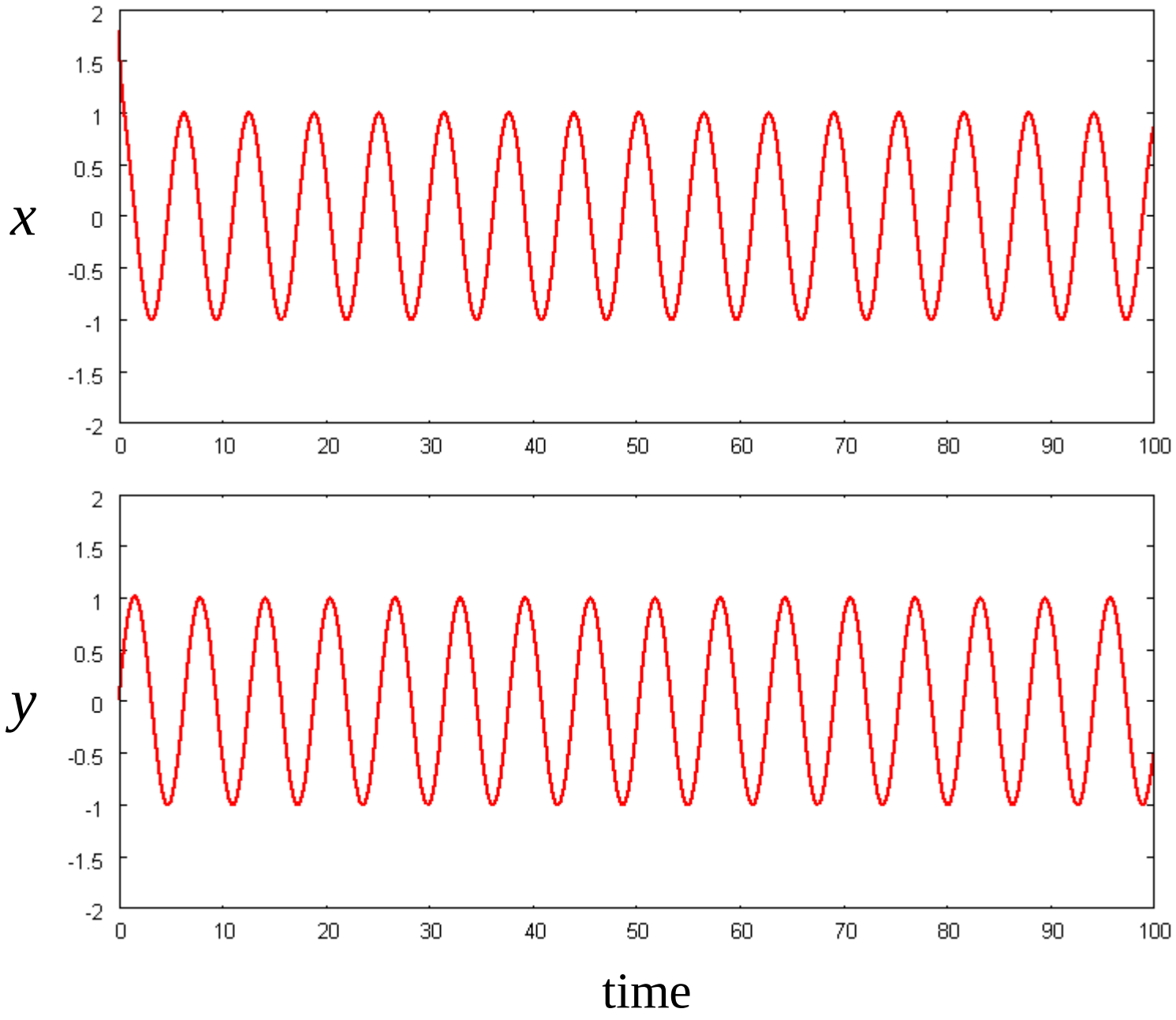
$$\begin{cases} \dfrac{\mathrm{d}r}{\mathrm{d}t} = ar - r^3 \\ \dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \omega \end{cases}$$

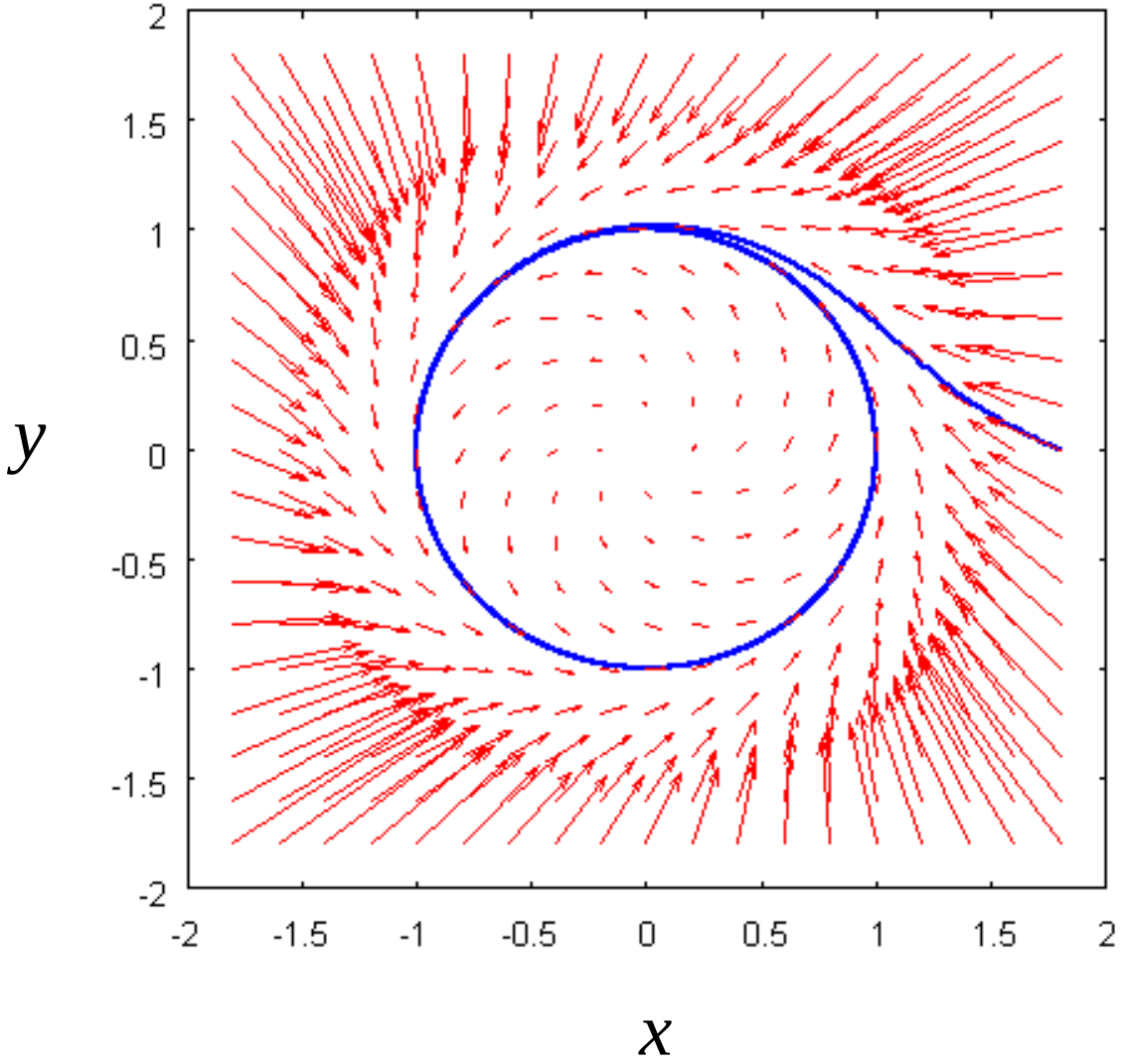
$$r^2 = x^2 + y^2$$

$$\frac{y}{x} = \tan\theta$$

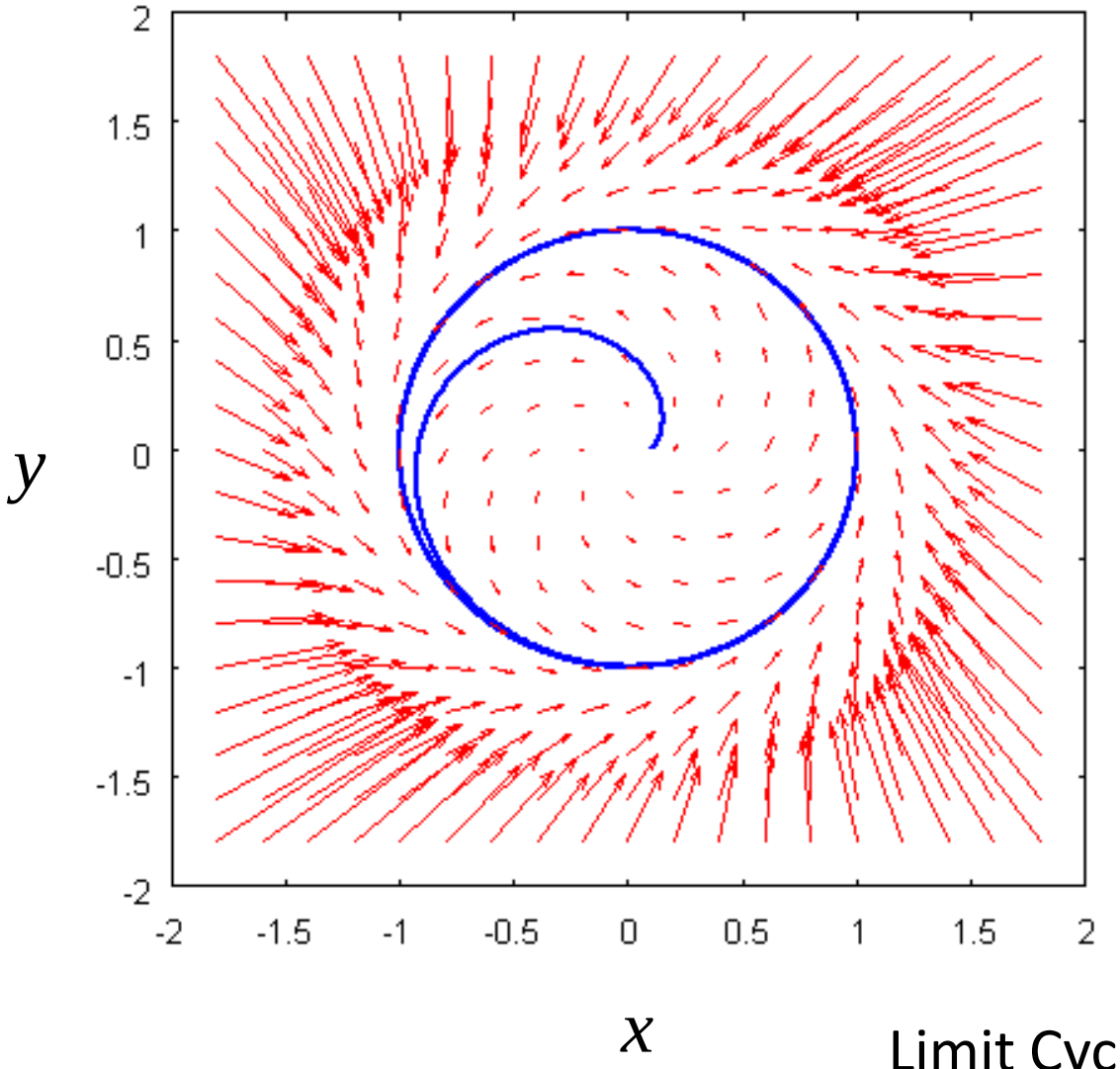
$$a = 1$$

$$b = 0$$


x
y
time
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
10
20
30
40
50
60
70
80
90
100

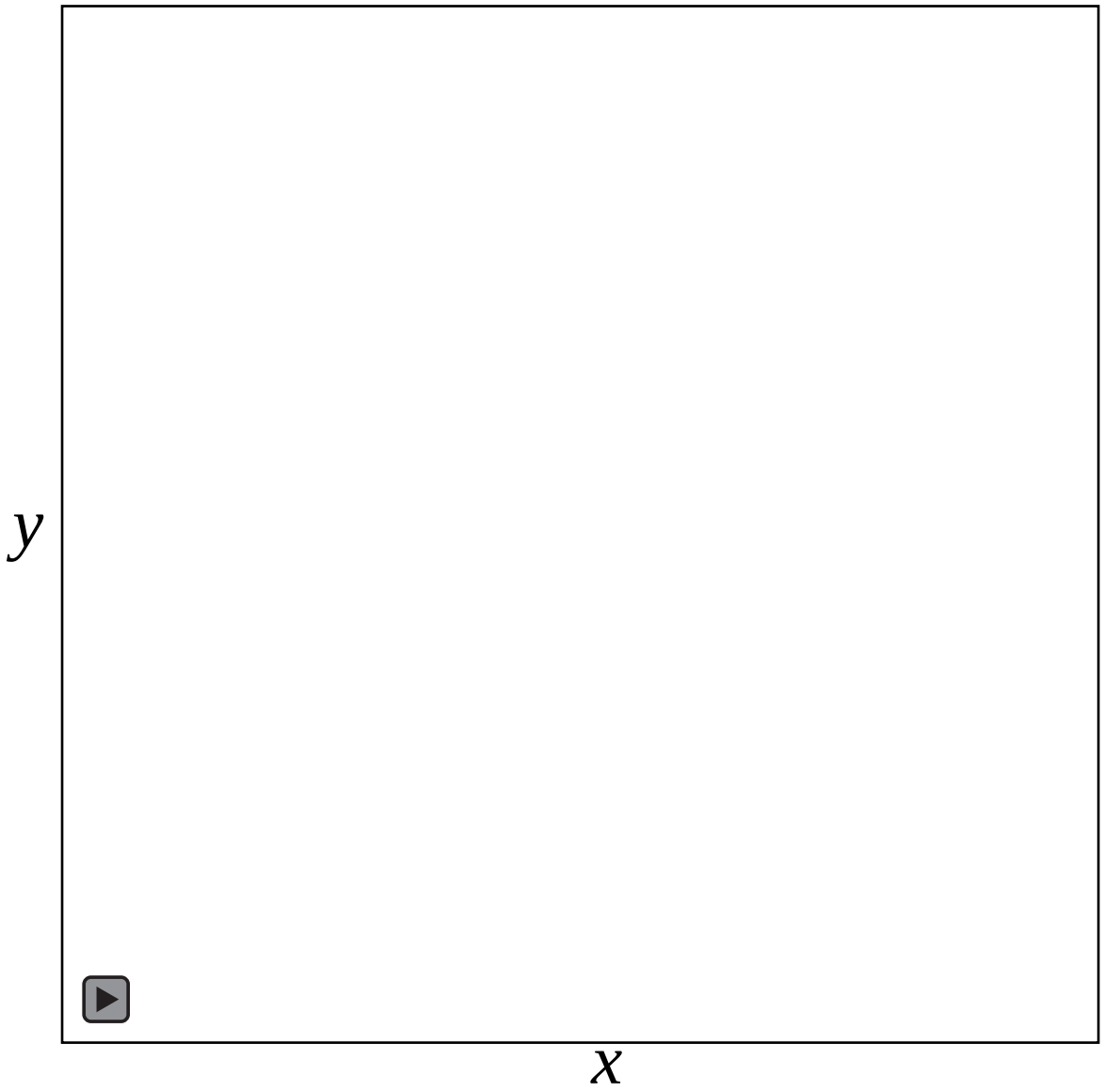

2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
y
x

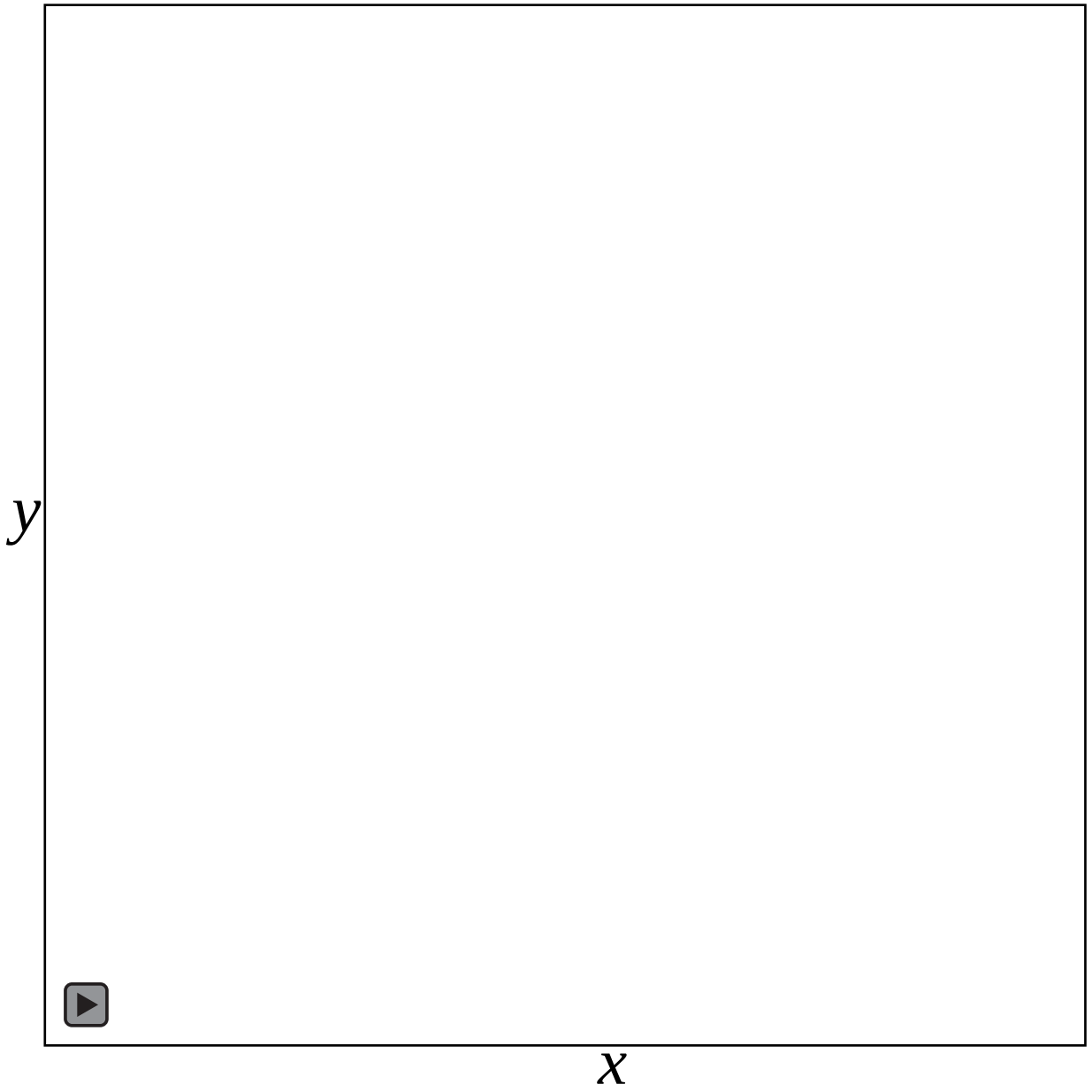
初期値を変えても

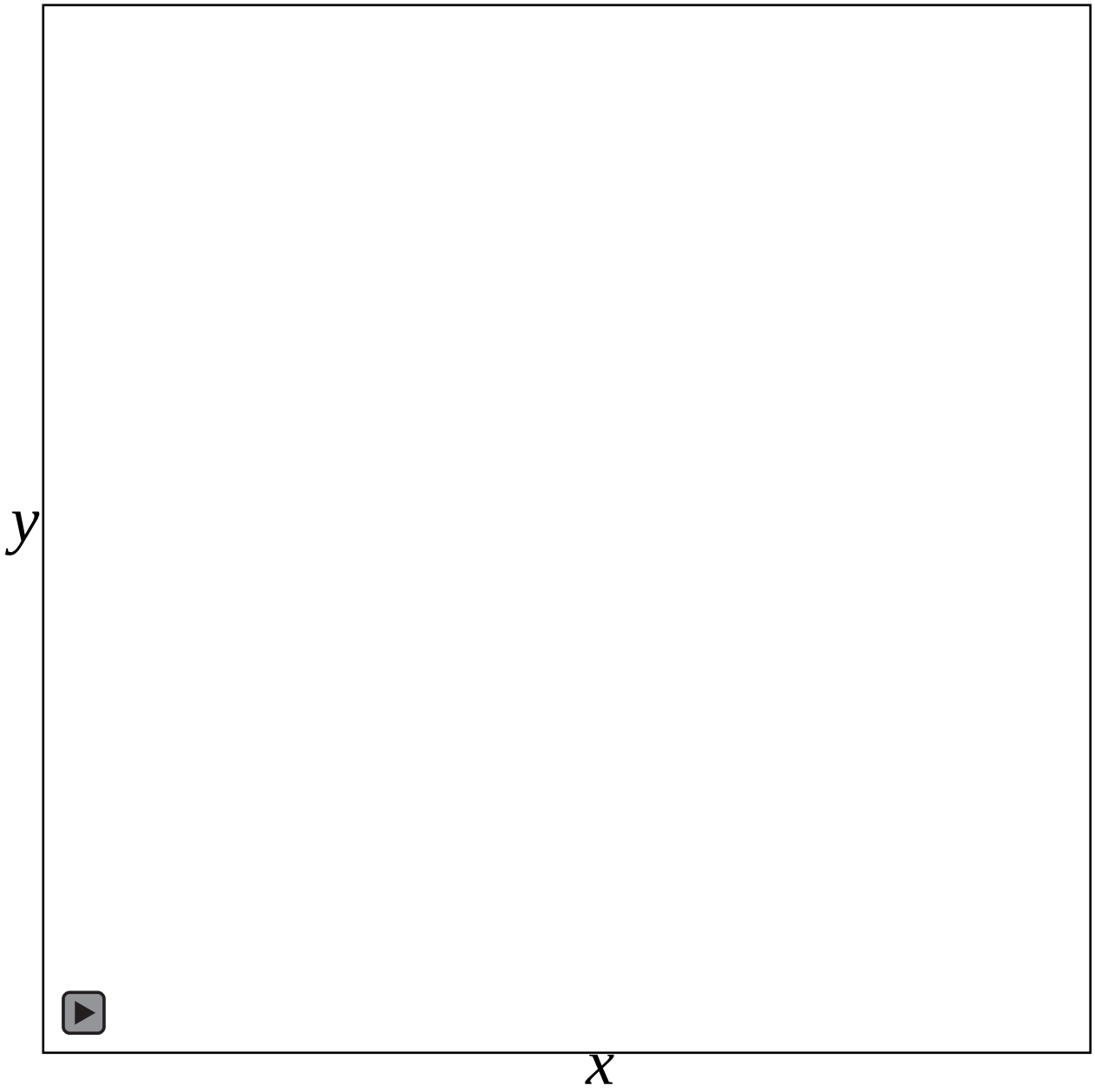


Limit Cycle (極限軌道)

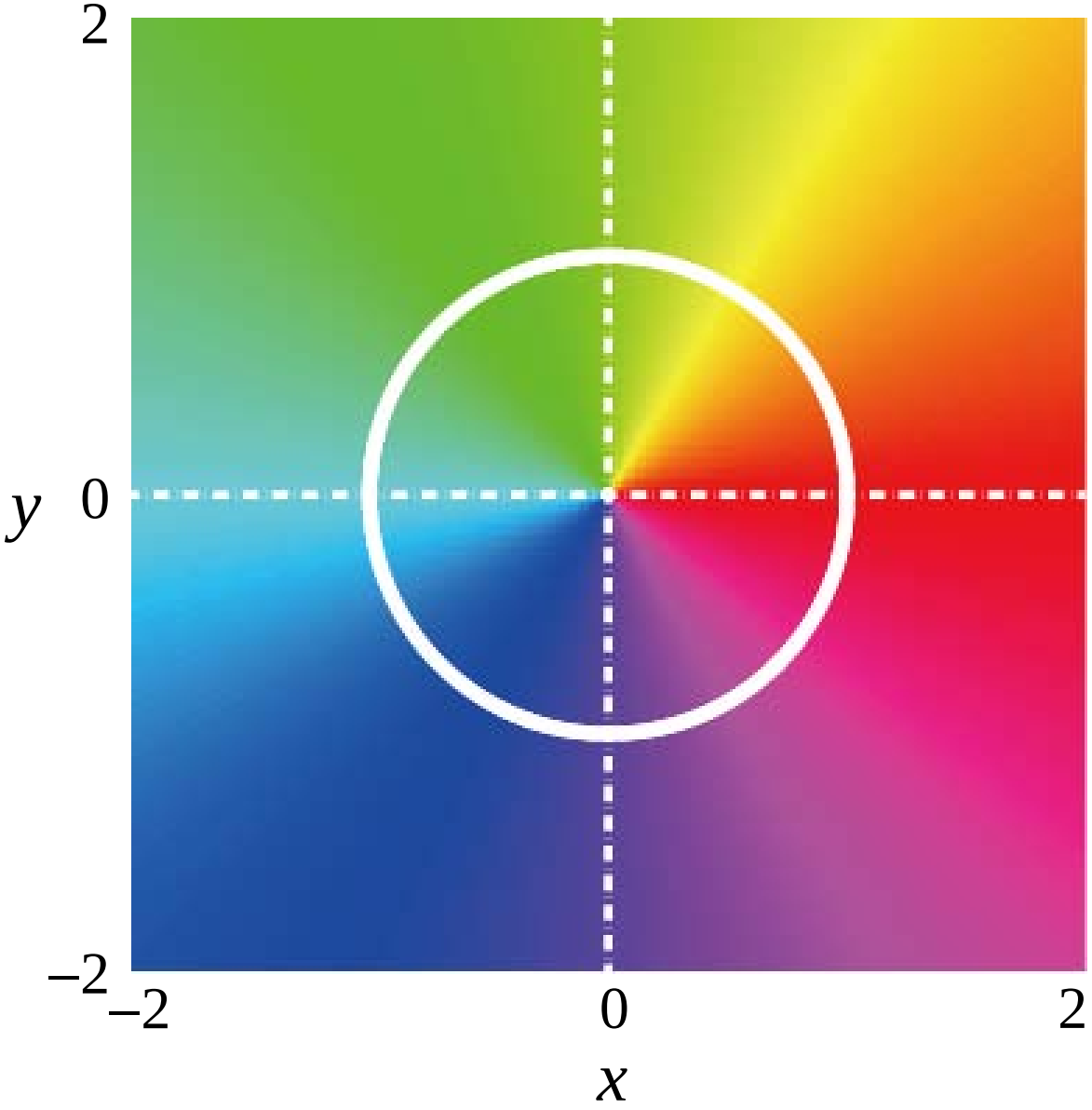
リミットサイクル上の運動




y
x


y
x

等位相面

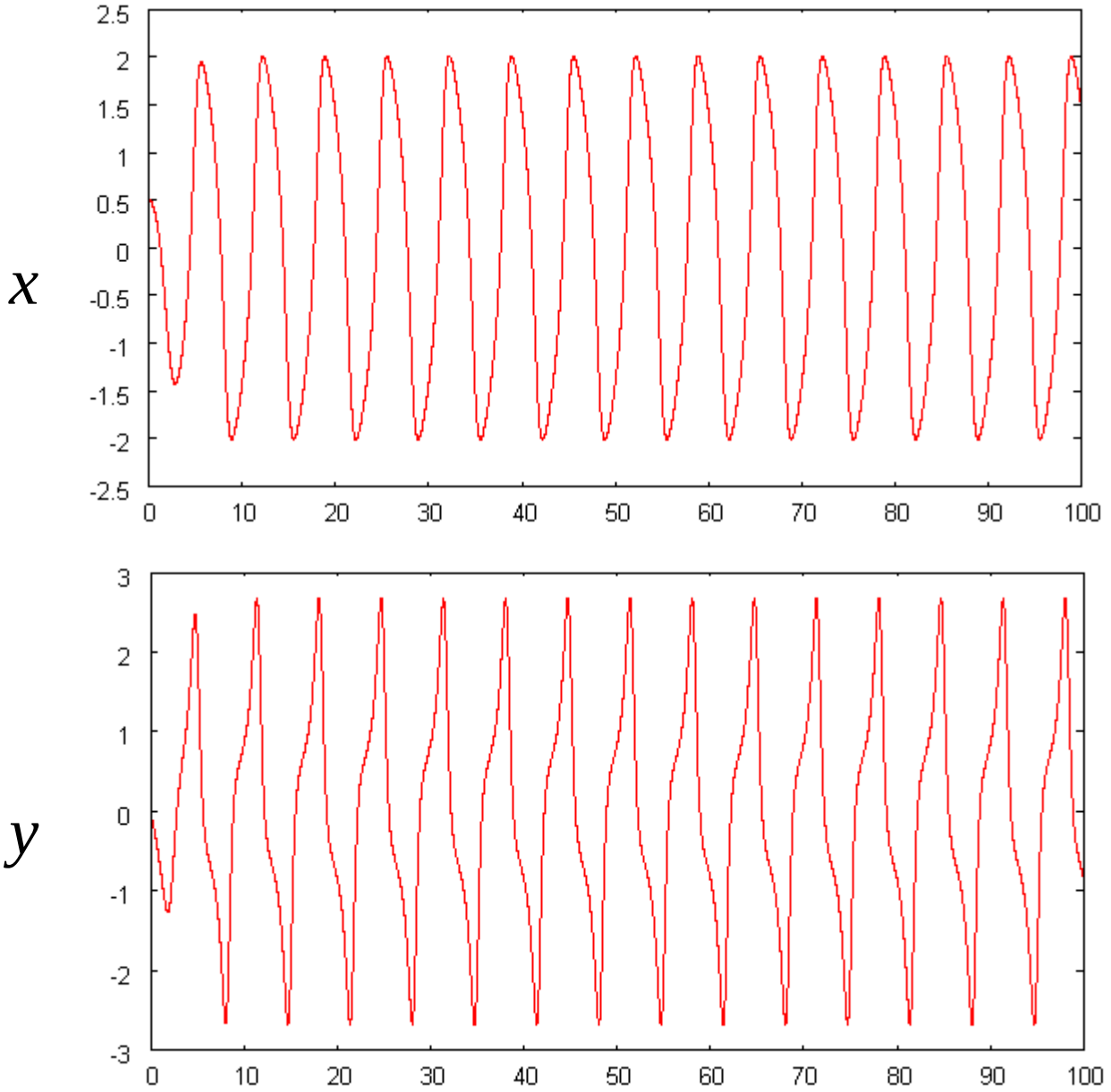


$$\theta = \arctan\left(\frac{y}{x}\right)$$

van der Pol 方程式　～　丸くなくても・・・

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \alpha\left(x^2 - 1\right)\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0$$

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = y \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -\alpha\left(x^2 - 1\right)y - x \end{cases}$$


x
y
time
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
3
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0
-1
-2
-3
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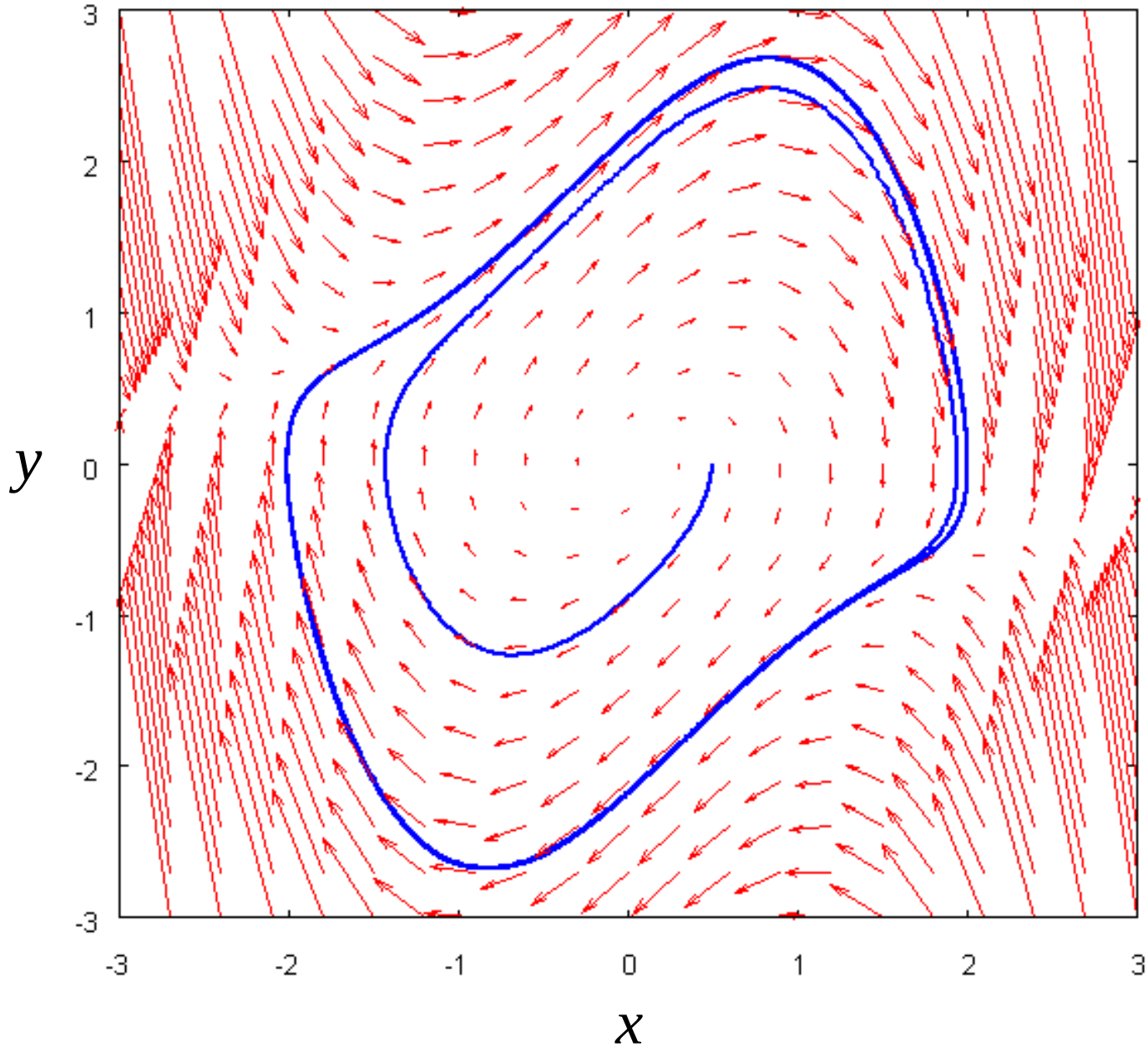
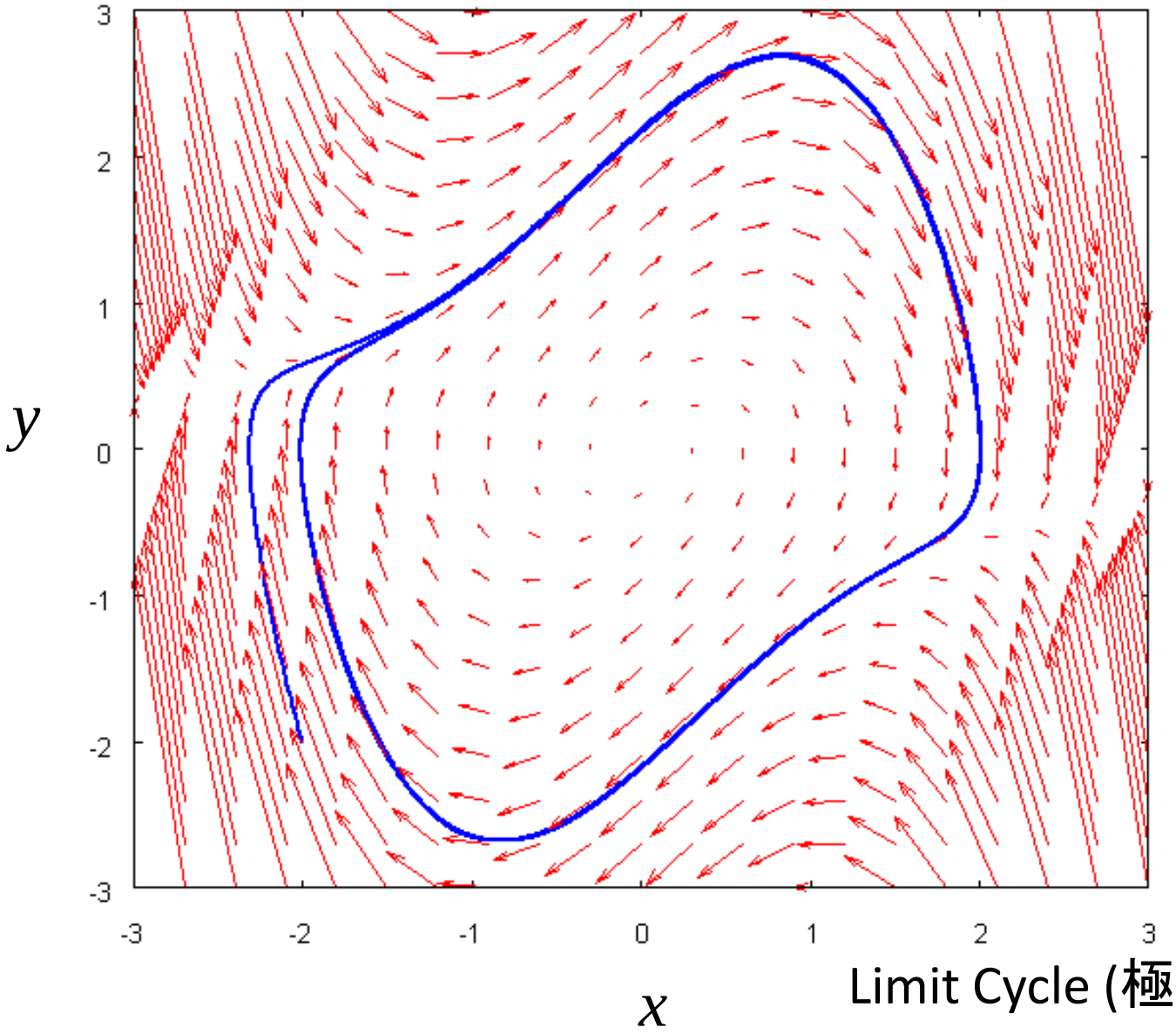
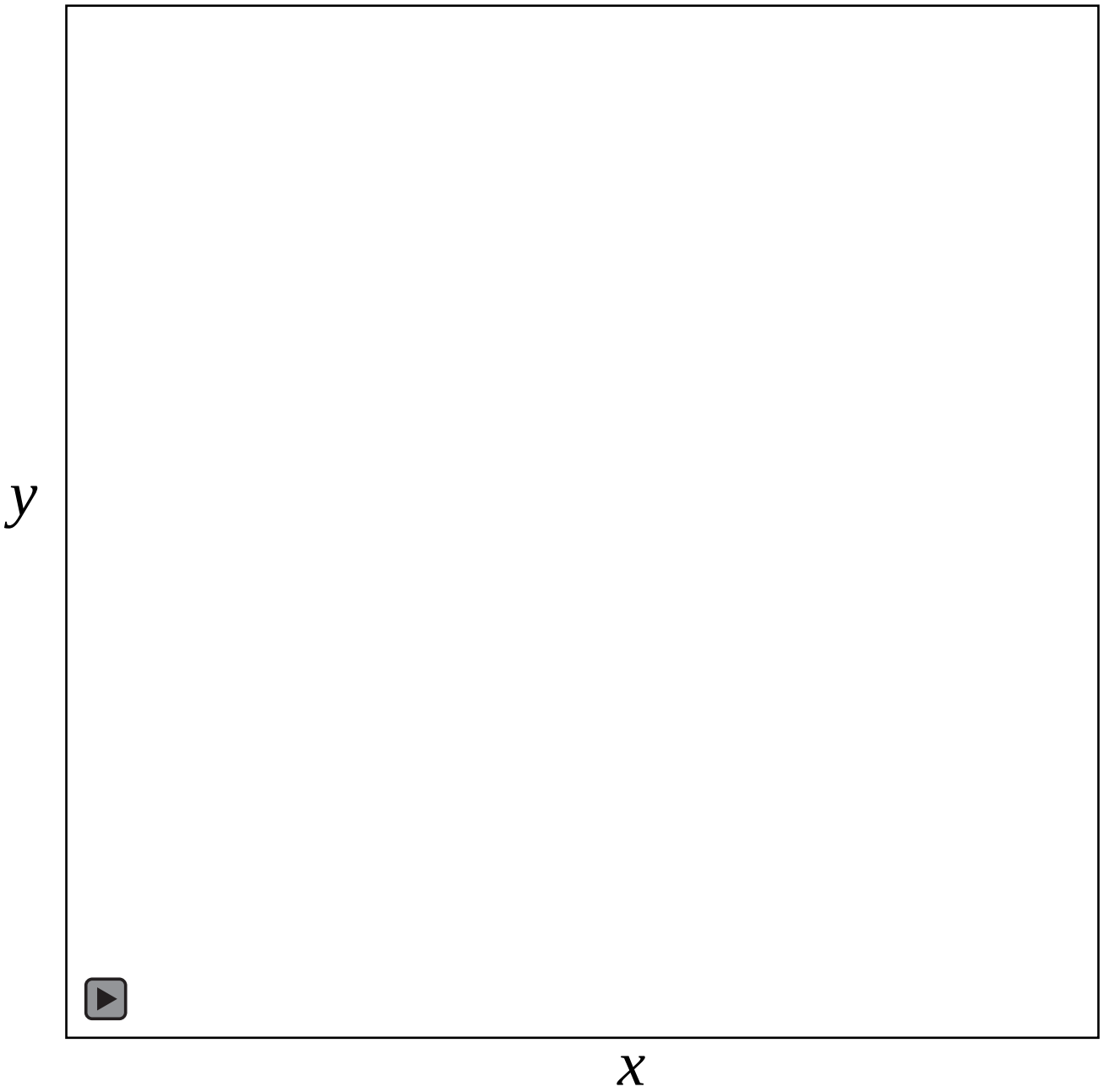
10
20
30
40
50
60
70
80
90
100

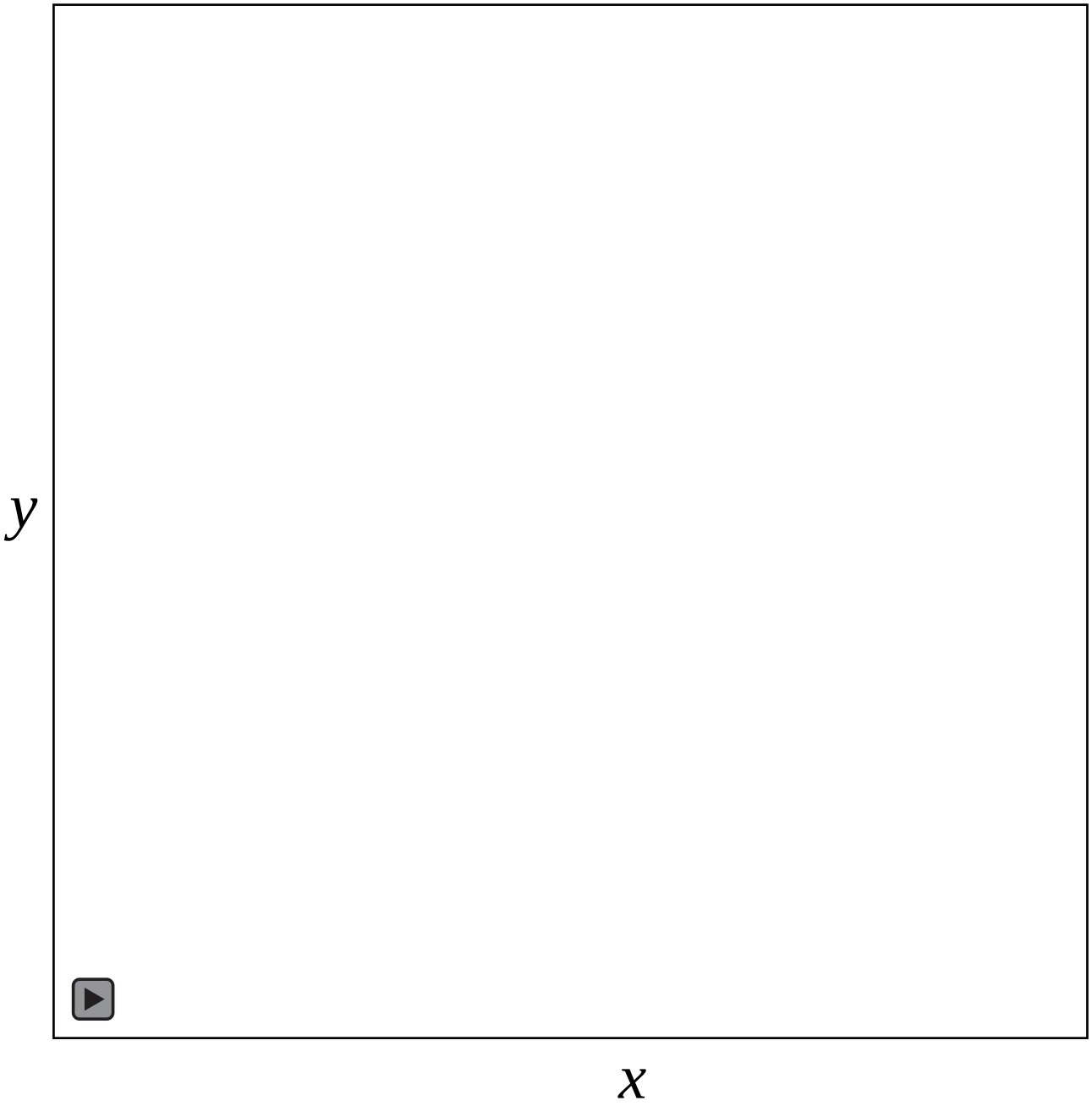

3
2
1
0
-1
-2
-3
y
-3
-2
-1
0
1
2
3
x

初期値を変えても

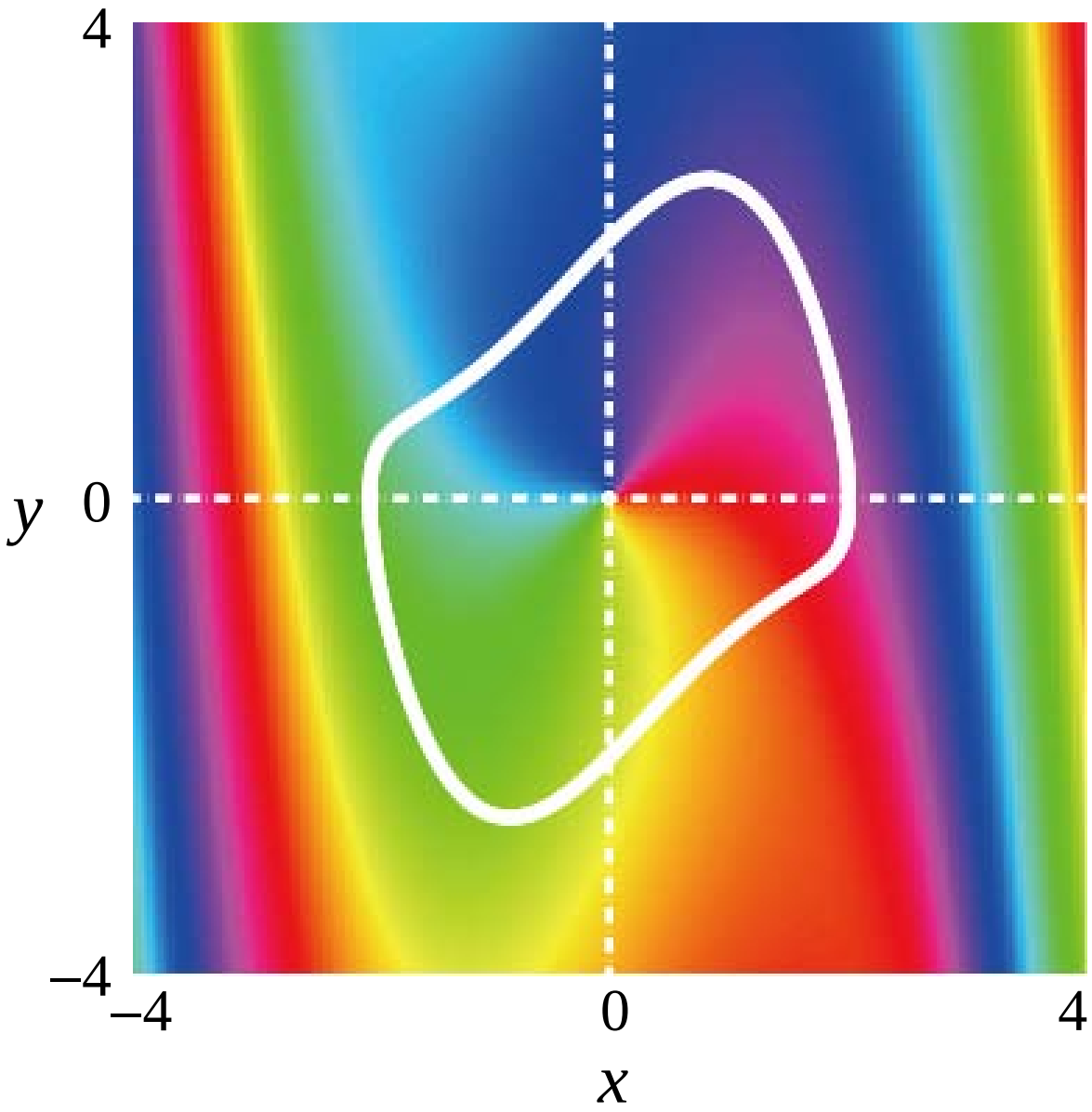


Limit Cycle (極限軌道)


y
x


y
x

等位相面



さまざまな非線形振動子

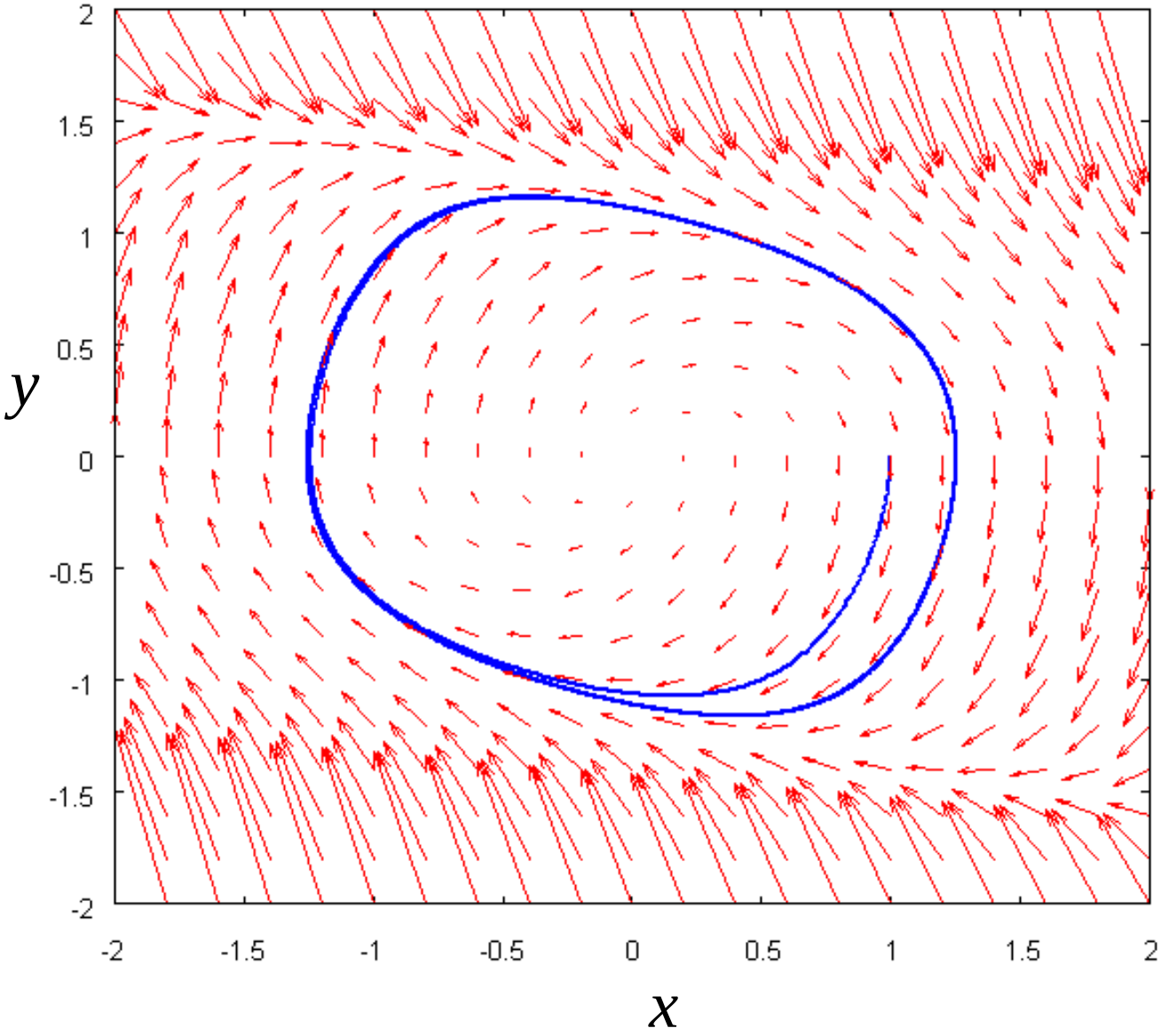
Rayleigh方程式

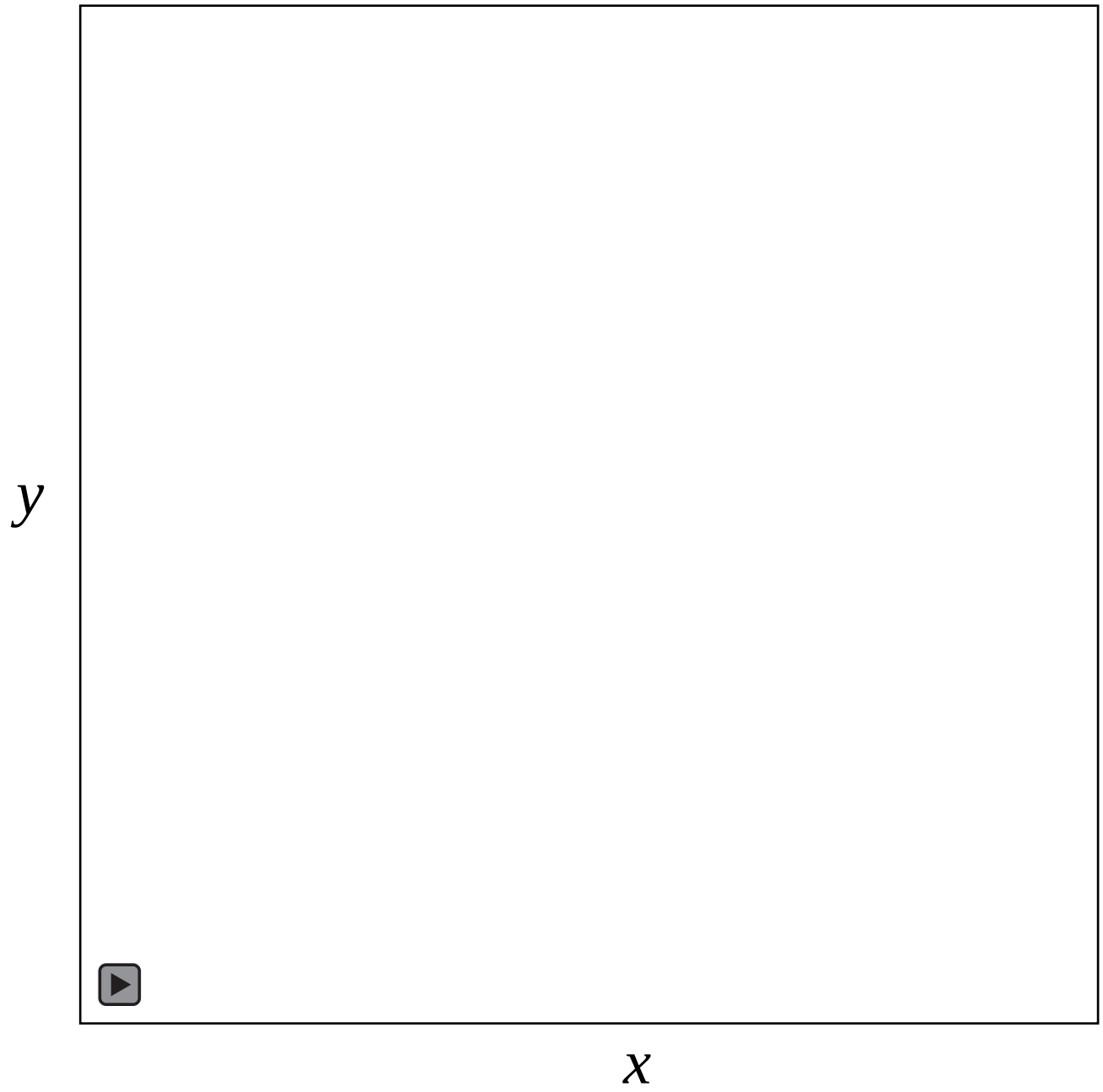
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \alpha\left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 - 1\right)\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0$$

FitzHugh-Nagumo方程式

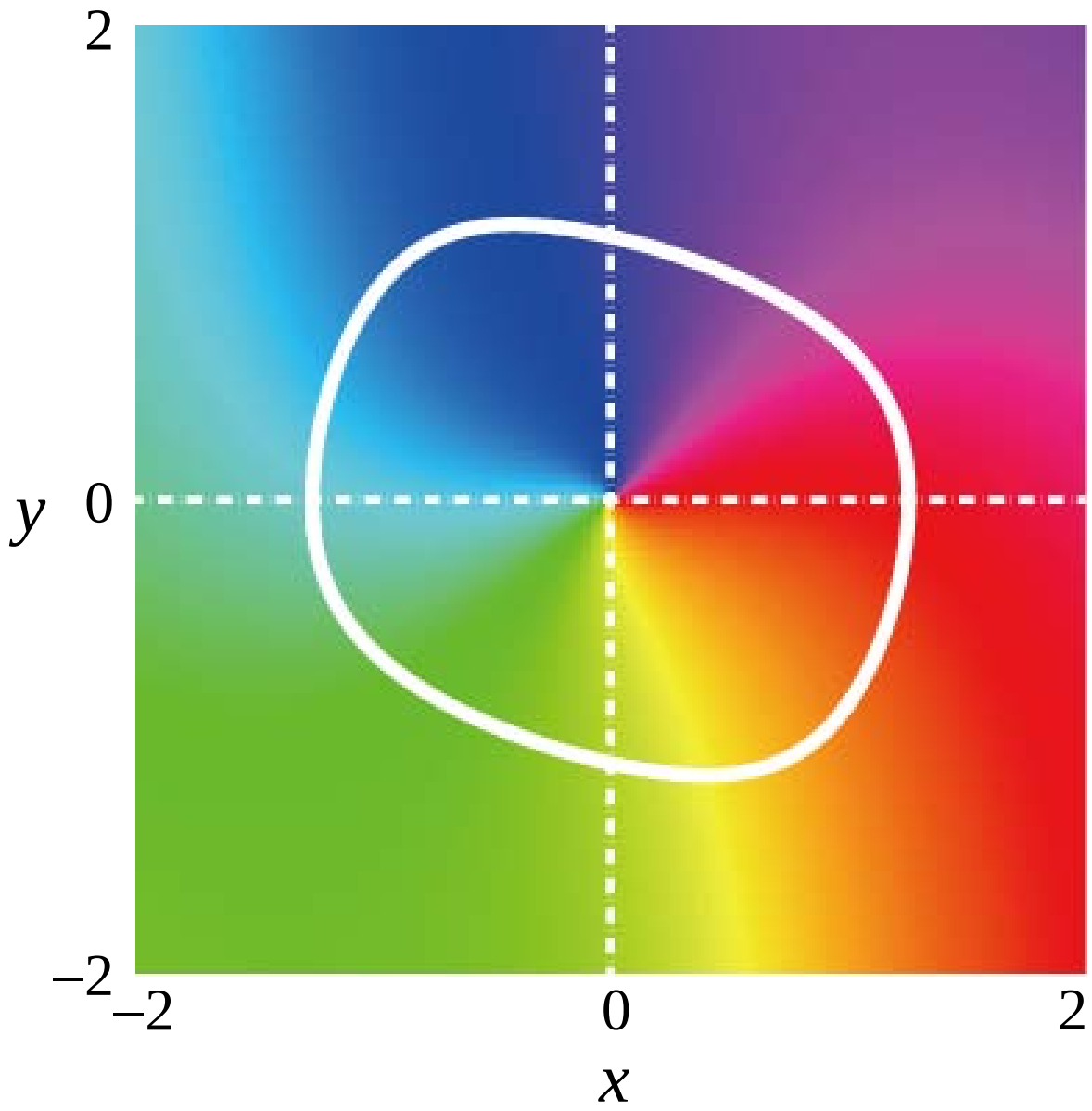
$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{1}{\varepsilon}\left(x - x^3 - y\right) \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = x - y + b \end{cases}$$

Rayleigh方程式

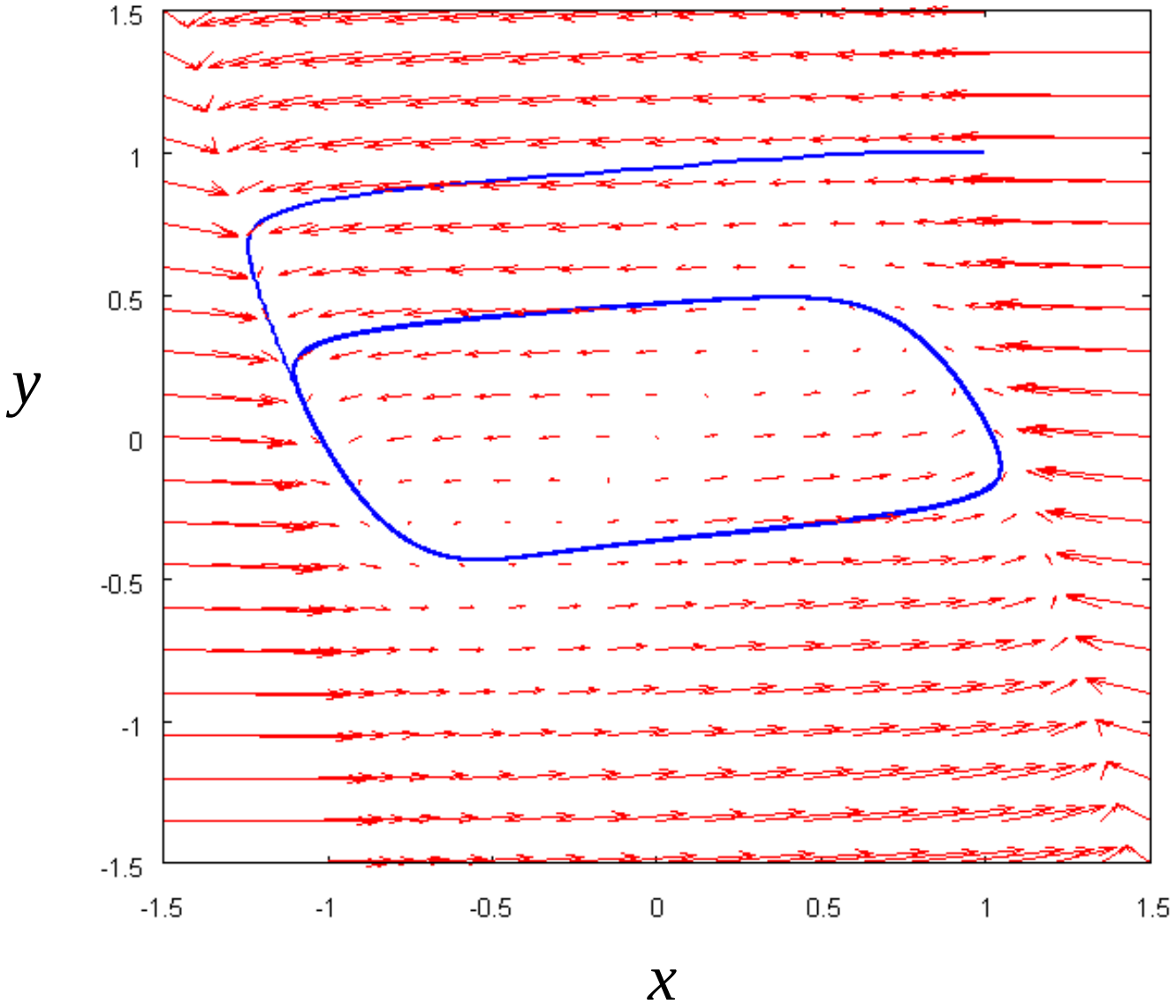


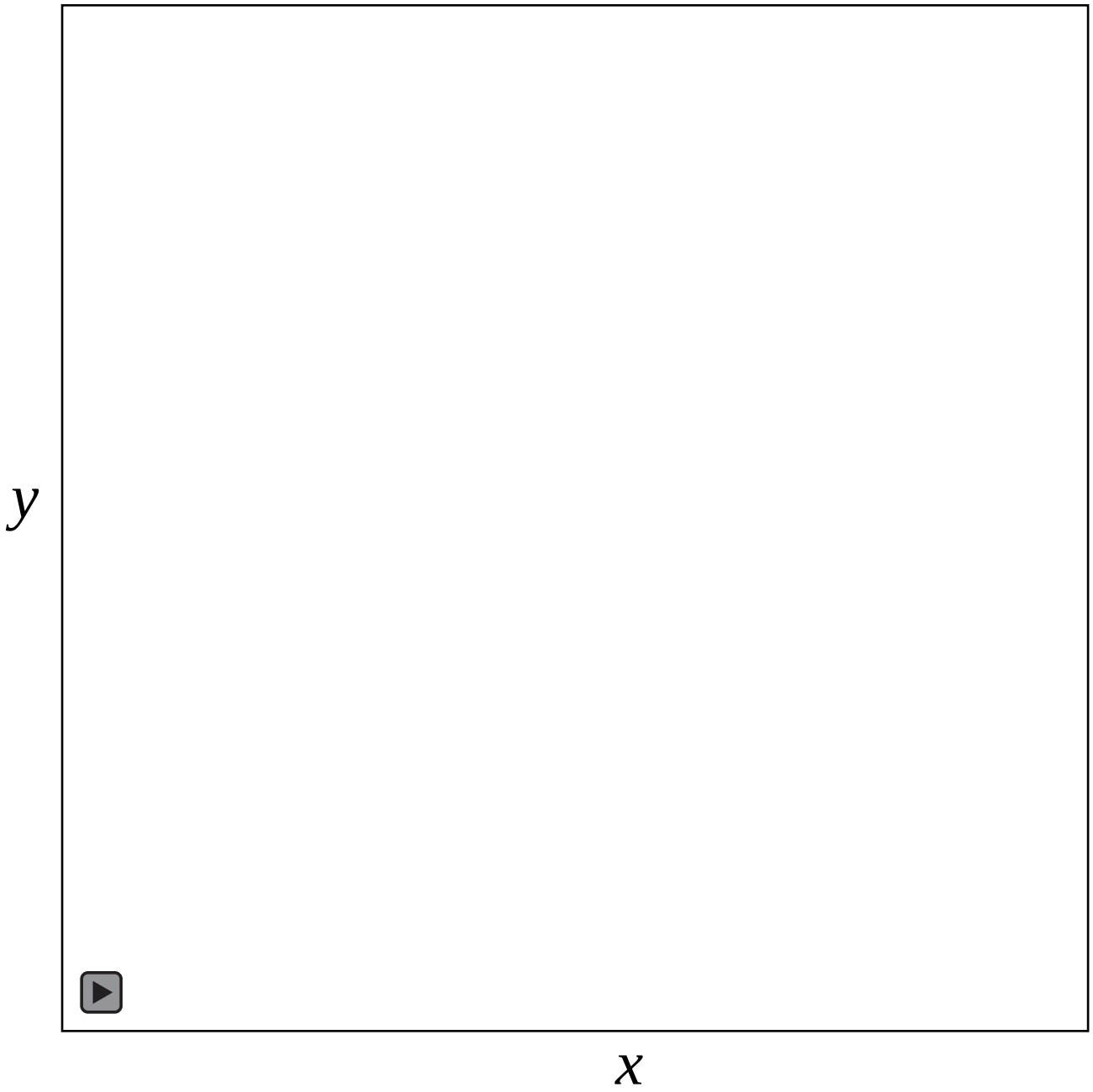

y
x

等位相面

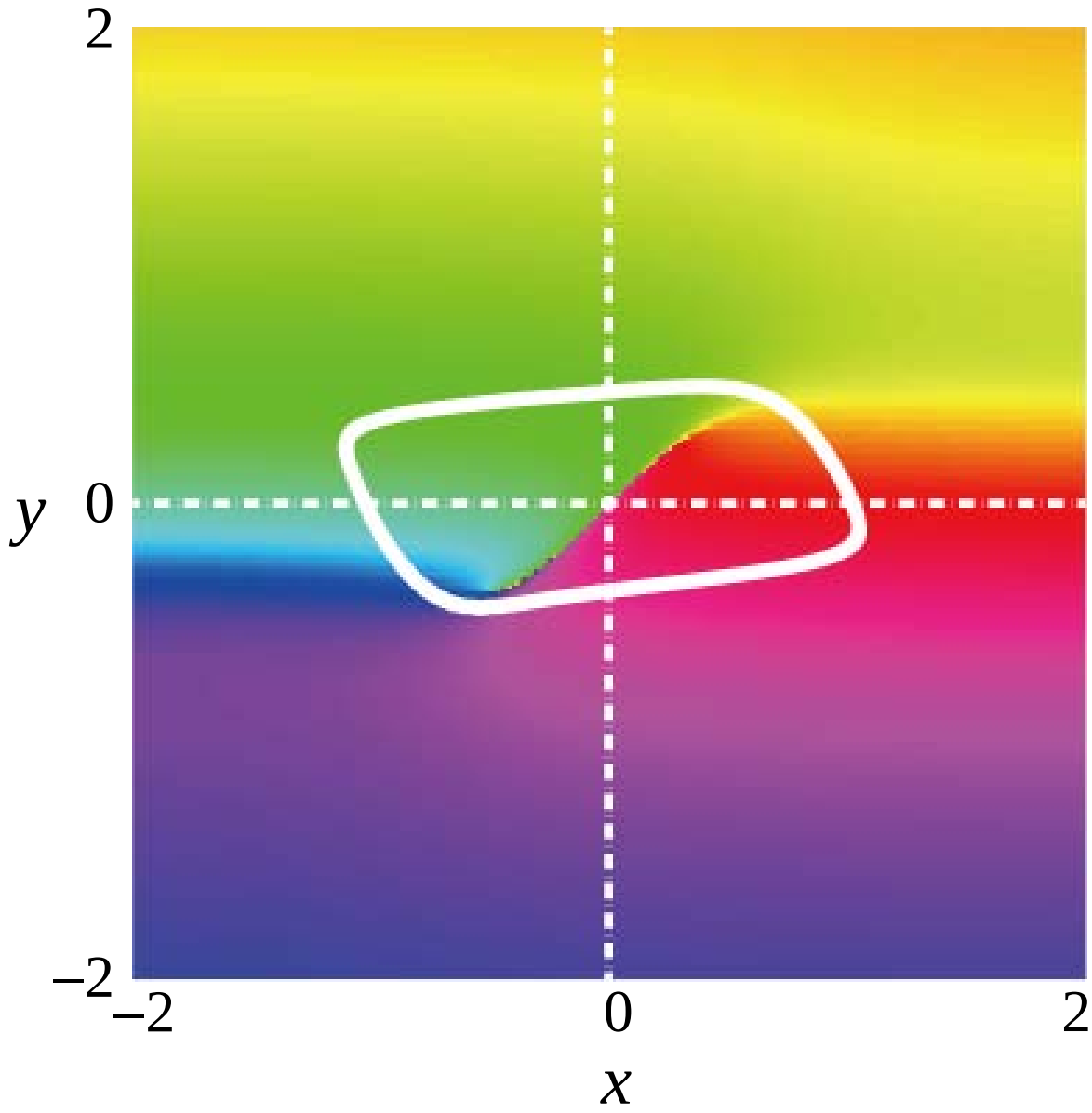


FitzHugh-Nagumo方程式



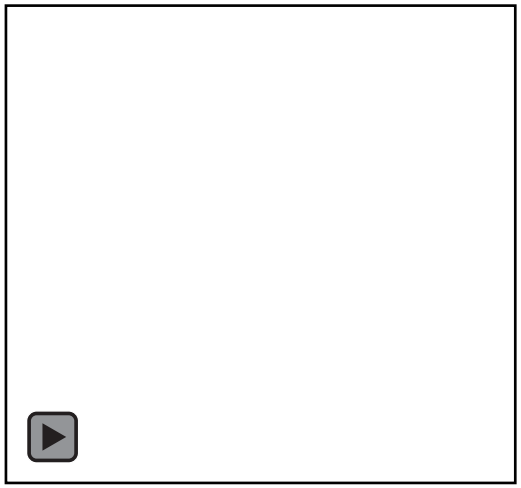

y
x

等位相面

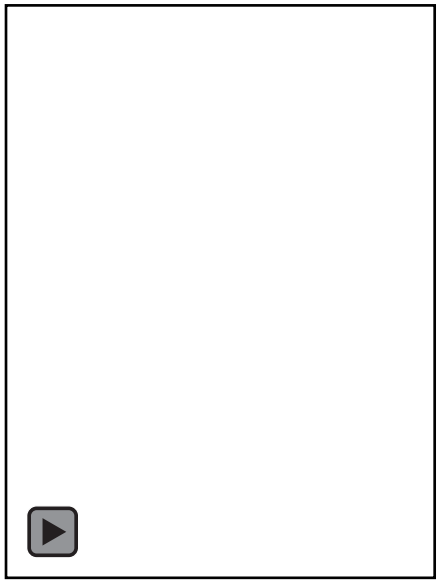


実験室で見られる時空間秩序形成

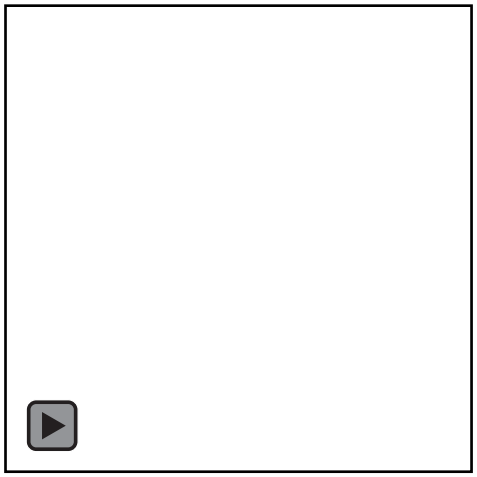
さまざまなリズム現象



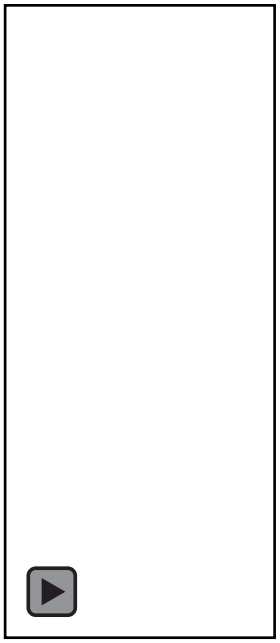
candle oscillator



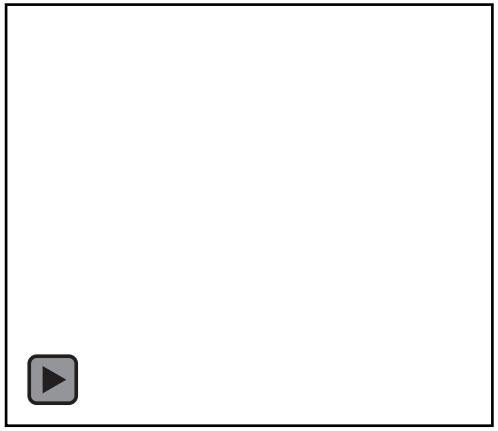
BR reaction



saline oscillator



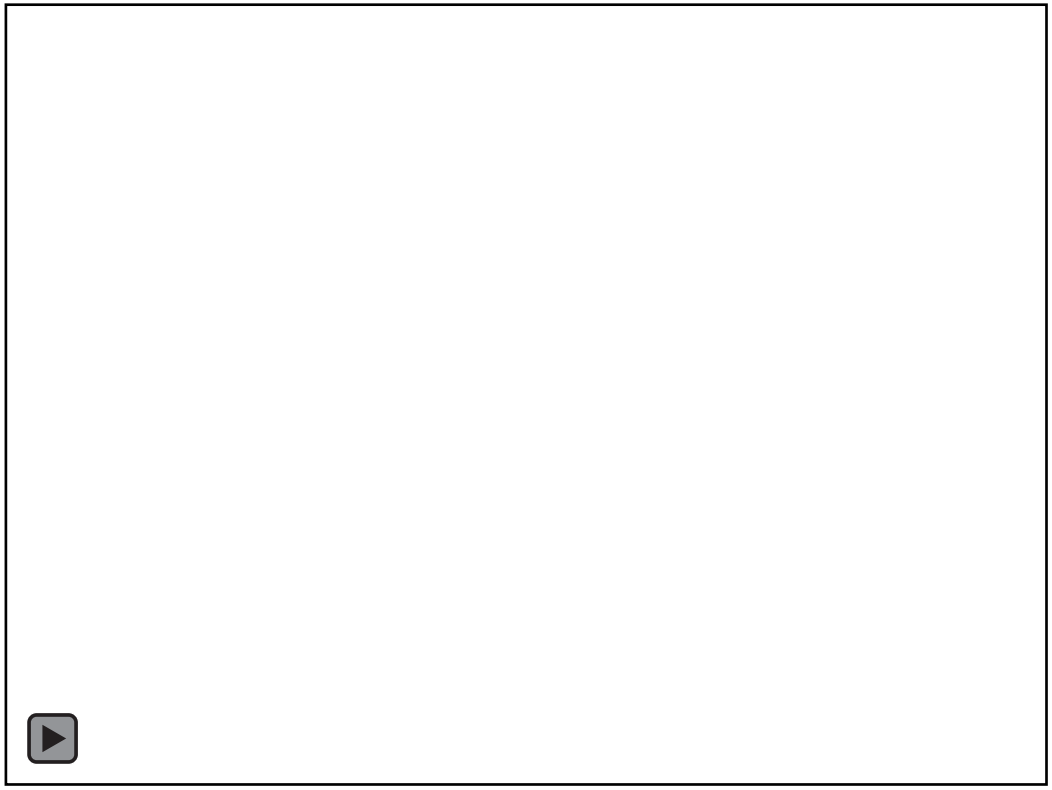
Plastic-bottle oscillator



water-camphor system

Belousov-Zhabotinsky (BZ)反応の実験

攪拌した系で

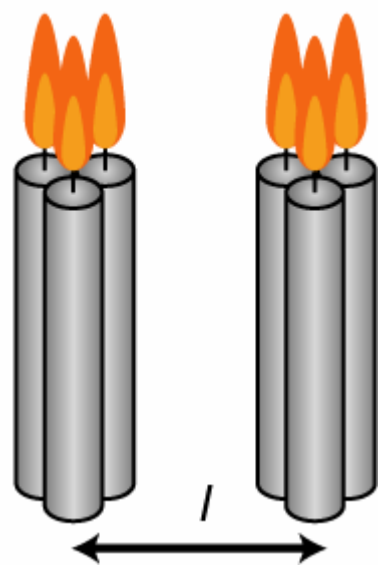


1 cm

空間勾配 はなし

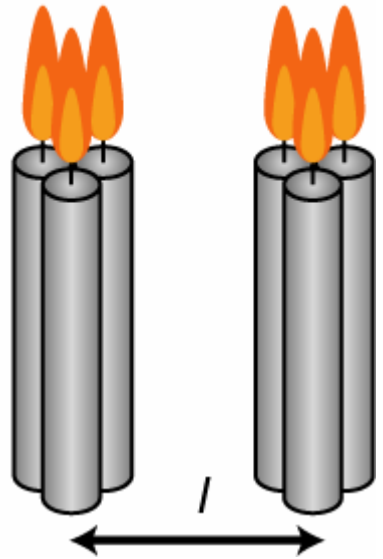
ろうそく

l = 2.0 mm



15/100 times fast

l = 4.0 mm

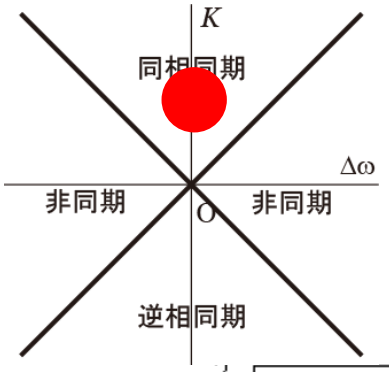


15/100 times fast

非線形振動子の結合系

2振動子の結合系

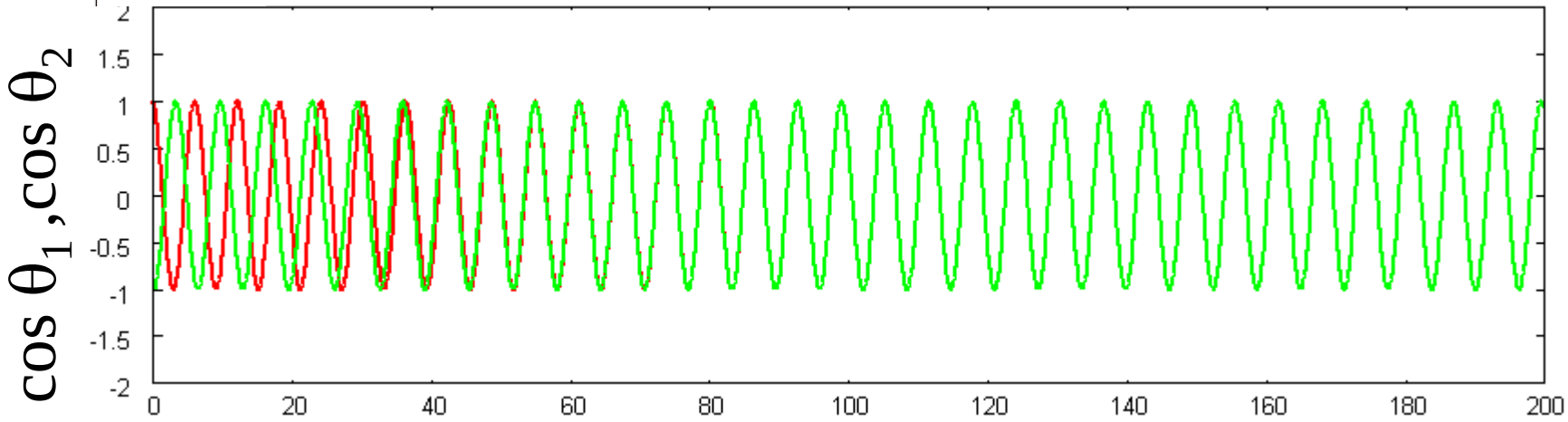
$$\begin{cases} \dfrac{\mathrm{d}\theta_1}{\mathrm{d}t} = \omega_1 + K\sin\left(\theta_2 - \theta_1\right) \\ \dfrac{\mathrm{d}\theta_2}{\mathrm{d}t} = \omega_2 + K\sin\left(\theta_1 - \theta_2\right) \end{cases}$$

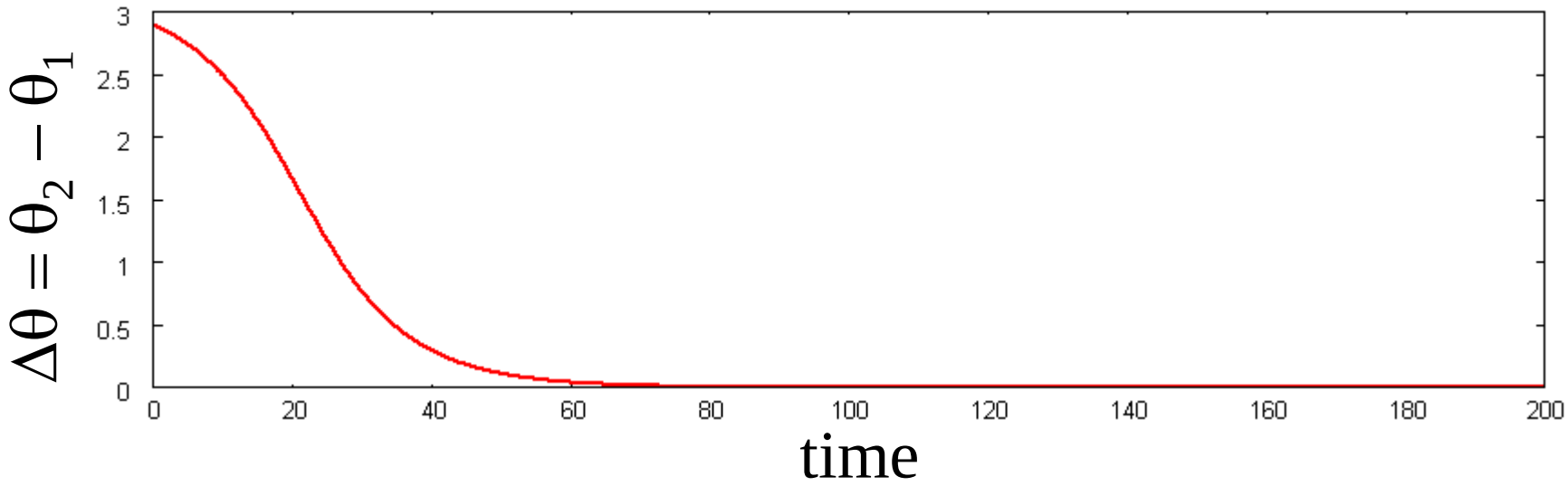


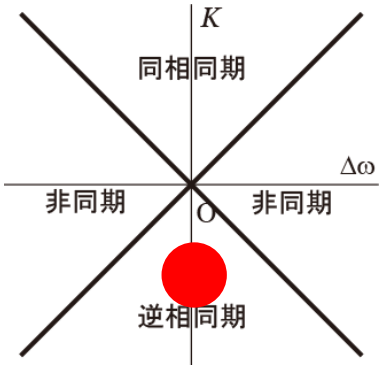
$$\begin{cases} \dfrac{\mathrm{d}\theta_1}{\mathrm{d}t} = \omega_1 + K\sin(\theta_2 - \theta_1) \\ \dfrac{\mathrm{d}\theta_2}{\mathrm{d}t} = \omega_2 + K\sin(\theta_1 - \theta_2) \end{cases}$$

$\omega_1 = \omega_2 = 1$

$K = 0.05$



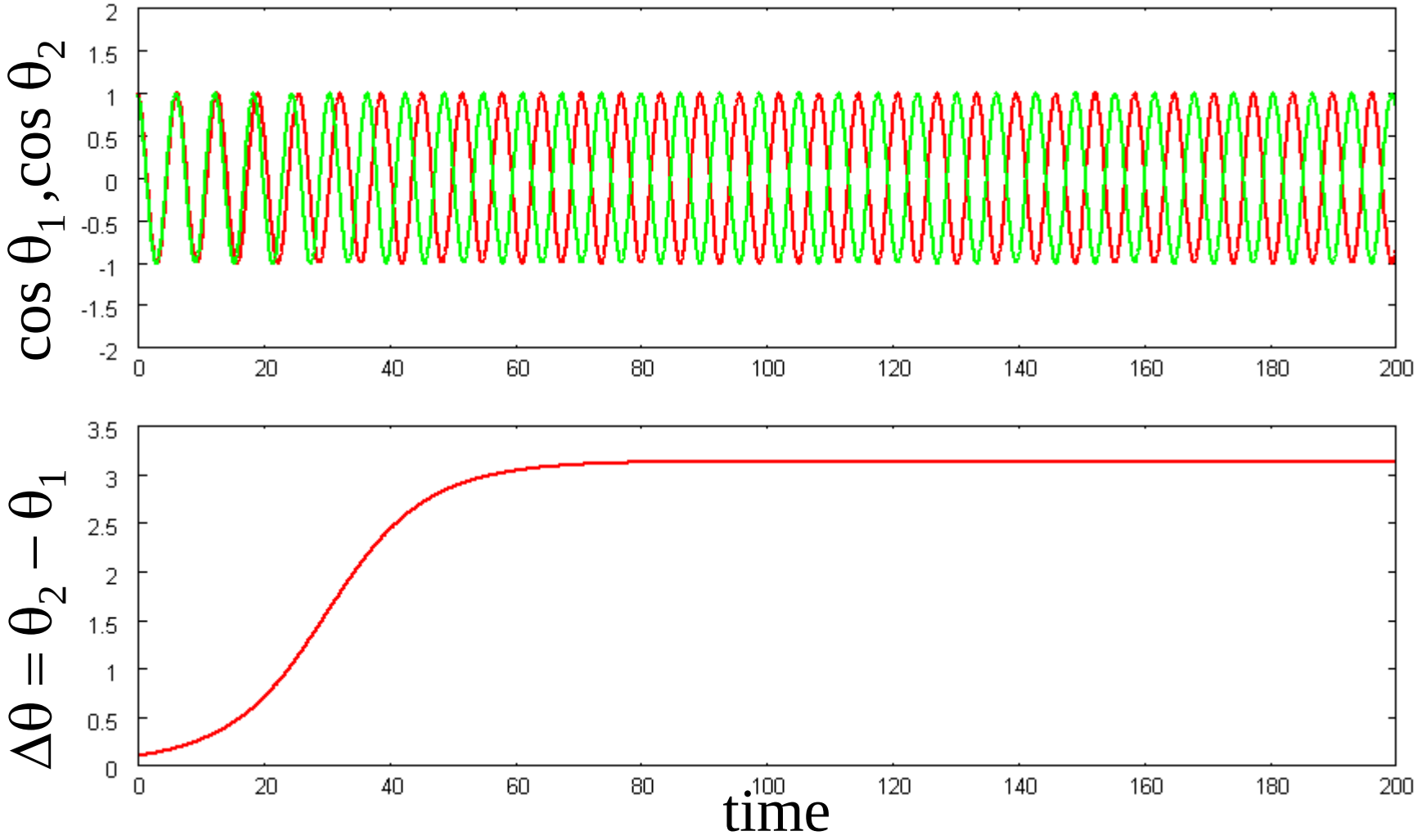


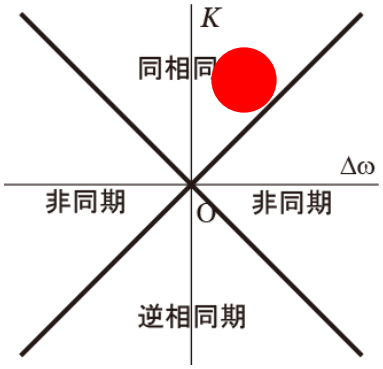


$$\begin{cases} \dfrac{d\theta_1}{dt} = \omega_1 + K\sin(\theta_2 - \theta_1) \\ \dfrac{d\theta_2}{dt} = \omega_2 + K\sin(\theta_1 - \theta_2) \end{cases}$$

$\omega_1 = \omega_2 = 1$

$K = -0.05$

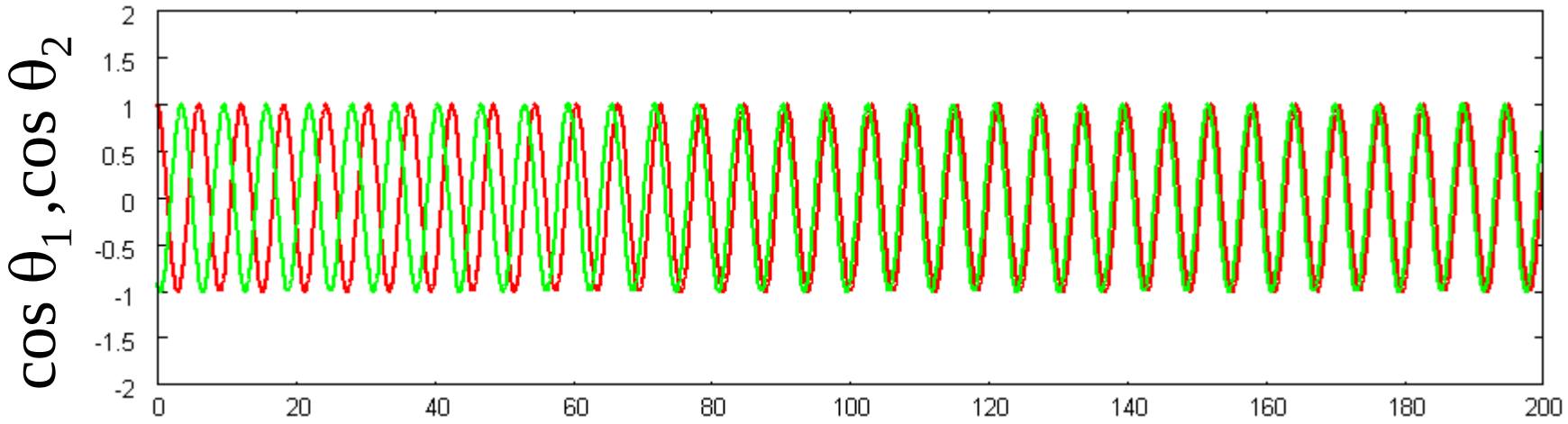


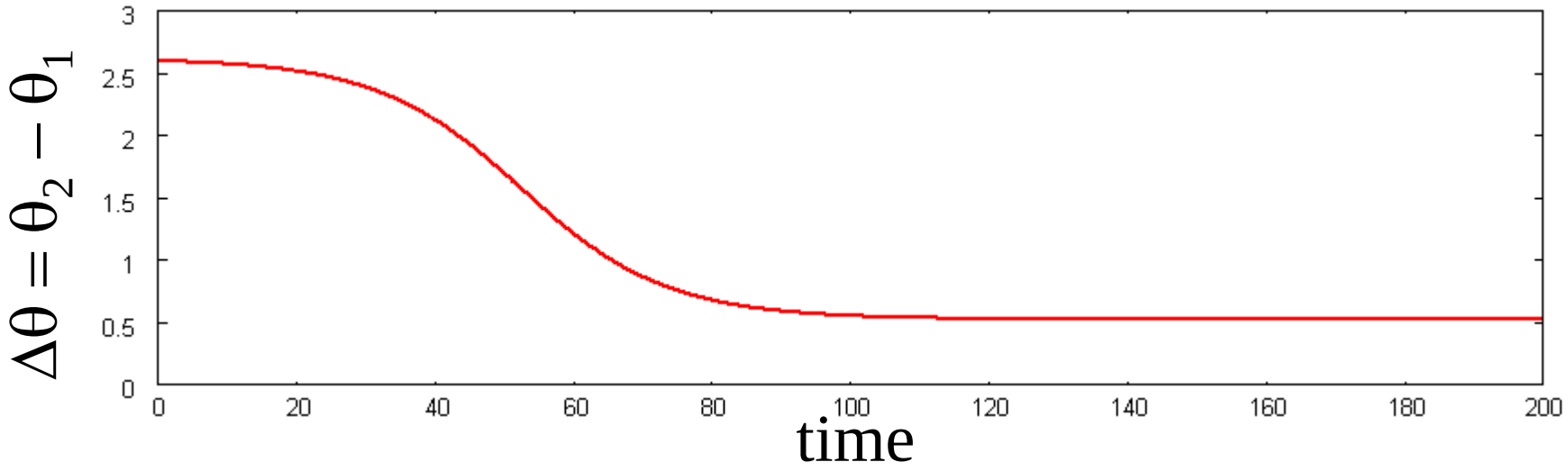


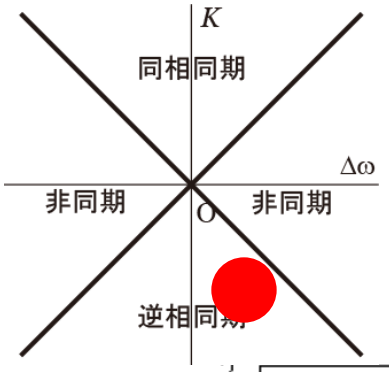
$$\begin{cases} \dfrac{\mathrm{d}\theta_1}{\mathrm{d}t} = \omega_1 + K\sin(\theta_2 - \theta_1) \\ \dfrac{\mathrm{d}\theta_2}{\mathrm{d}t} = \omega_2 + K\sin(\theta_1 - \theta_2) \end{cases}$$

ω_1=1, ω_2 = 1.05

K = 0.05



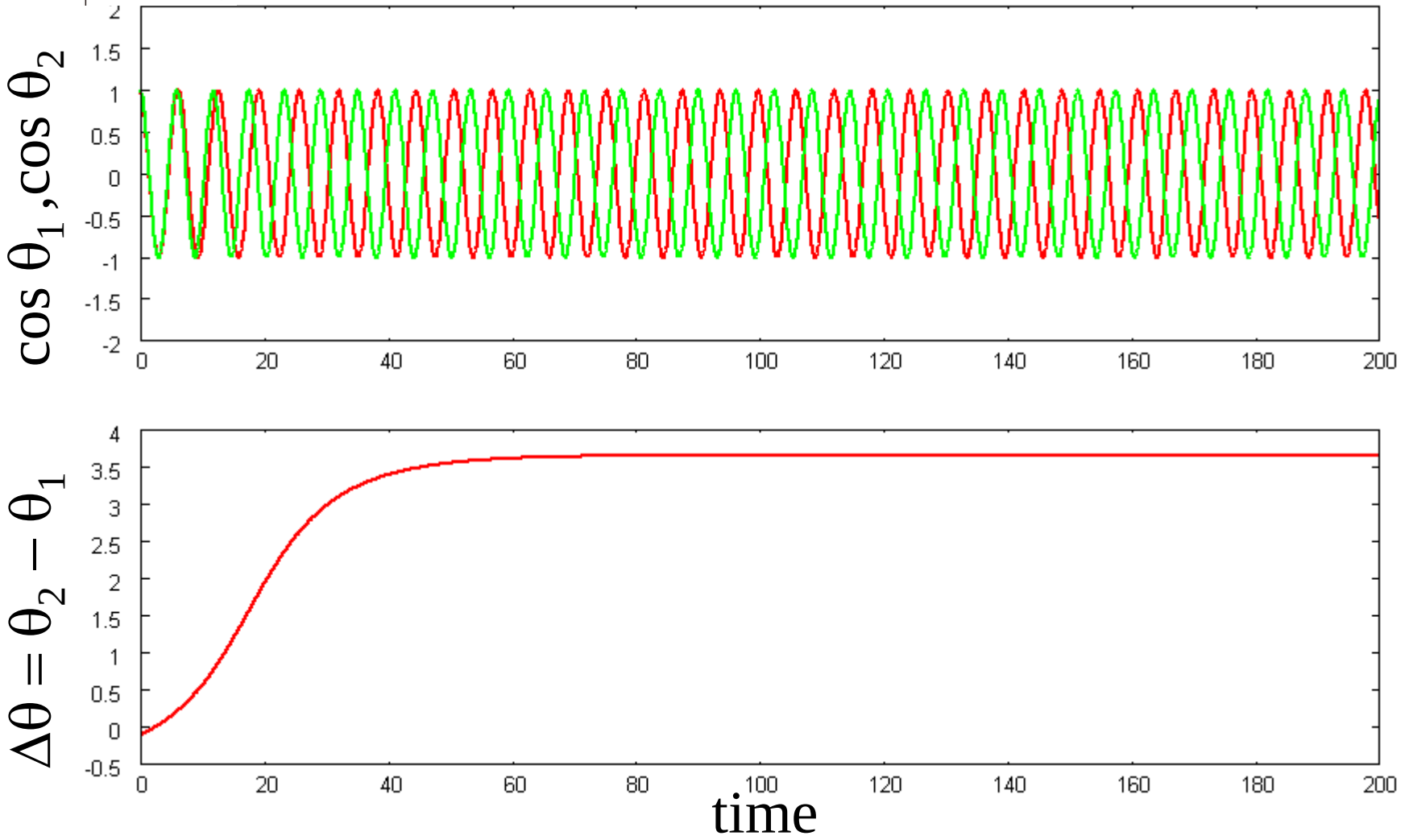


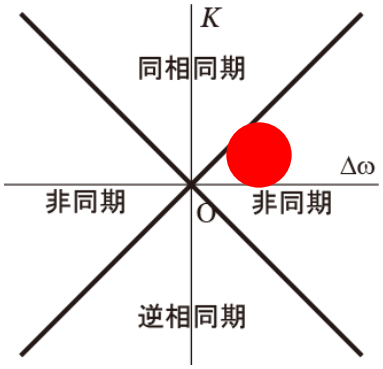


$$\begin{cases} \dfrac{d\theta_1}{dt} = \omega_1 + K\sin(\theta_2 - \theta_1) \\ \dfrac{d\theta_2}{dt} = \omega_2 + K\sin(\theta_1 - \theta_2) \end{cases}$$

$\omega_1 = 1,\ \omega_2 = 1.05$

$K = -0.05$

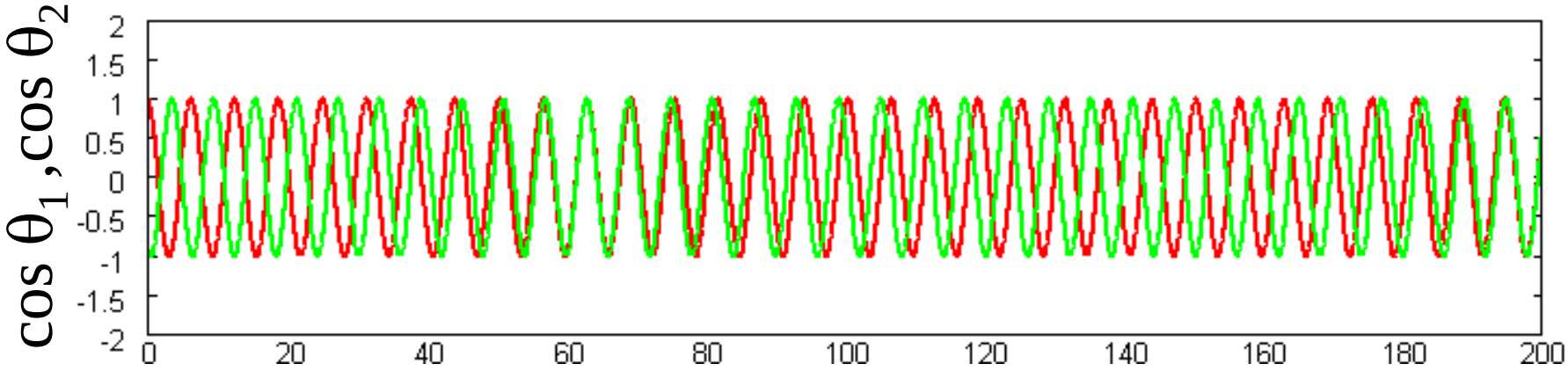


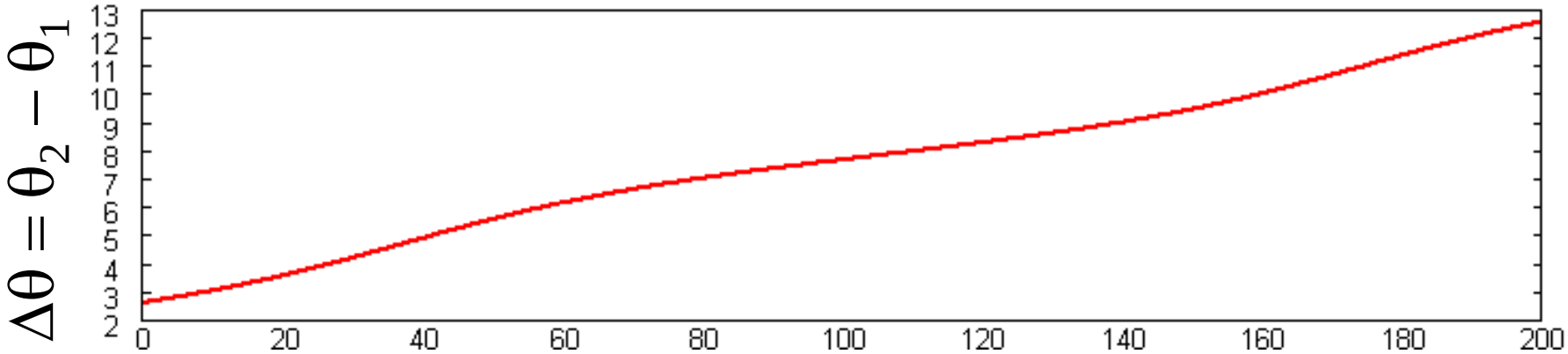


$$\begin{cases} \dfrac{\mathrm{d}\theta_1}{\mathrm{d}t} = \omega_1 + K\sin(\theta_2 - \theta_1) \\ \dfrac{\mathrm{d}\theta_2}{\mathrm{d}t} = \omega_2 + K\sin(\theta_1 - \theta_2) \end{cases}$$

$\omega_1 = 1,\ \omega_2 = 1.05$

$K = 0.01$





time