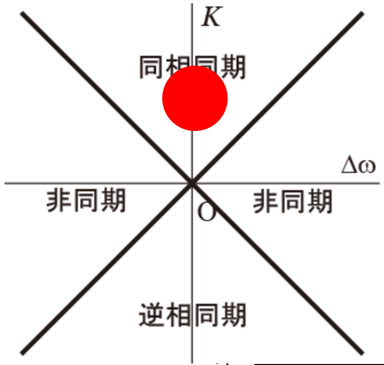
2013.12.17
物性物理学C

非線形振動子の結合系

非線形振動子の結合系

2振動子の結合系

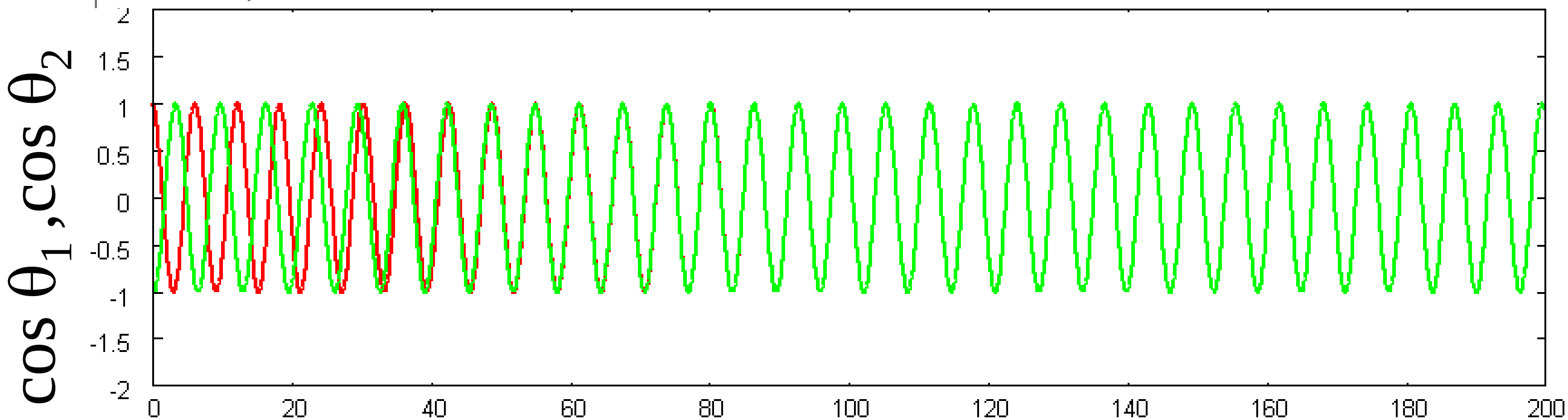
$$
\begin{cases}
\dfrac{\mathrm{d}\theta_1}{\mathrm{d}t} = \omega_1 + K\sin(\theta_2 - \theta_1) \\
\dfrac{\mathrm{d}\theta_2}{\mathrm{d}t} = \omega_2 + K\sin(\theta_1 - \theta_2)
\end{cases}
$$

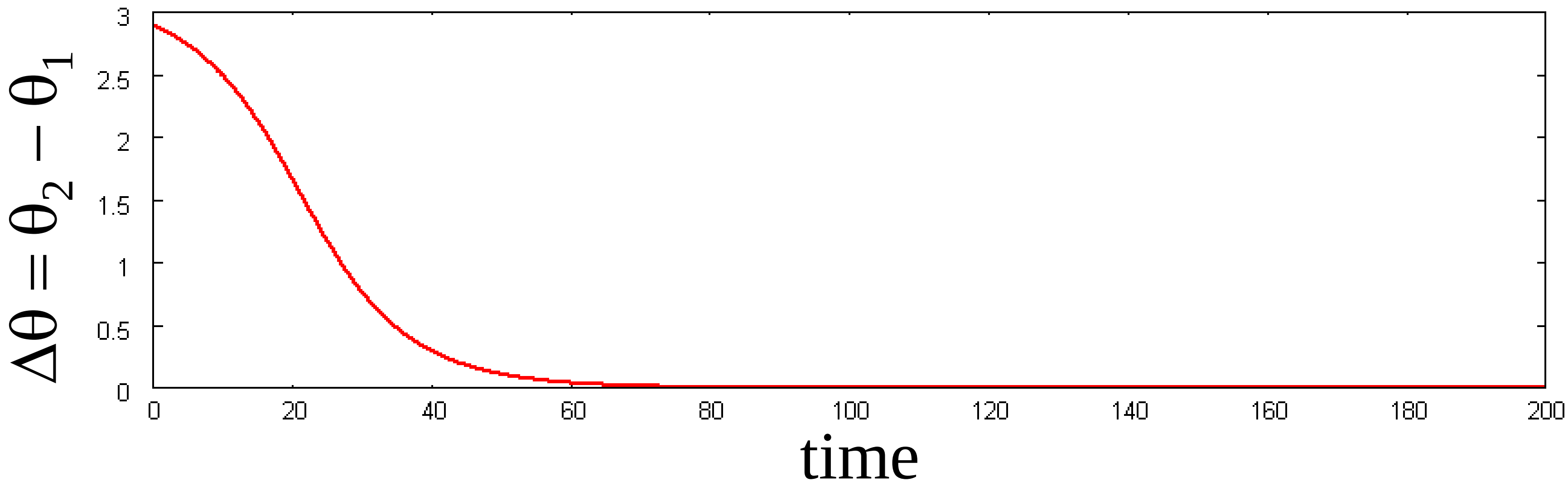


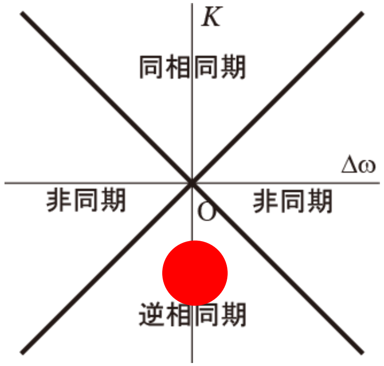
$$\begin{cases} \dfrac{d\theta_1}{dt} = \omega_1 + K\sin(\theta_2 - \theta_1) \\ \dfrac{d\theta_2}{dt} = \omega_2 + K\sin(\theta_1 - \theta_2) \end{cases}$$

$\omega_1 = \omega_2 = 1$

$K = 0.05$



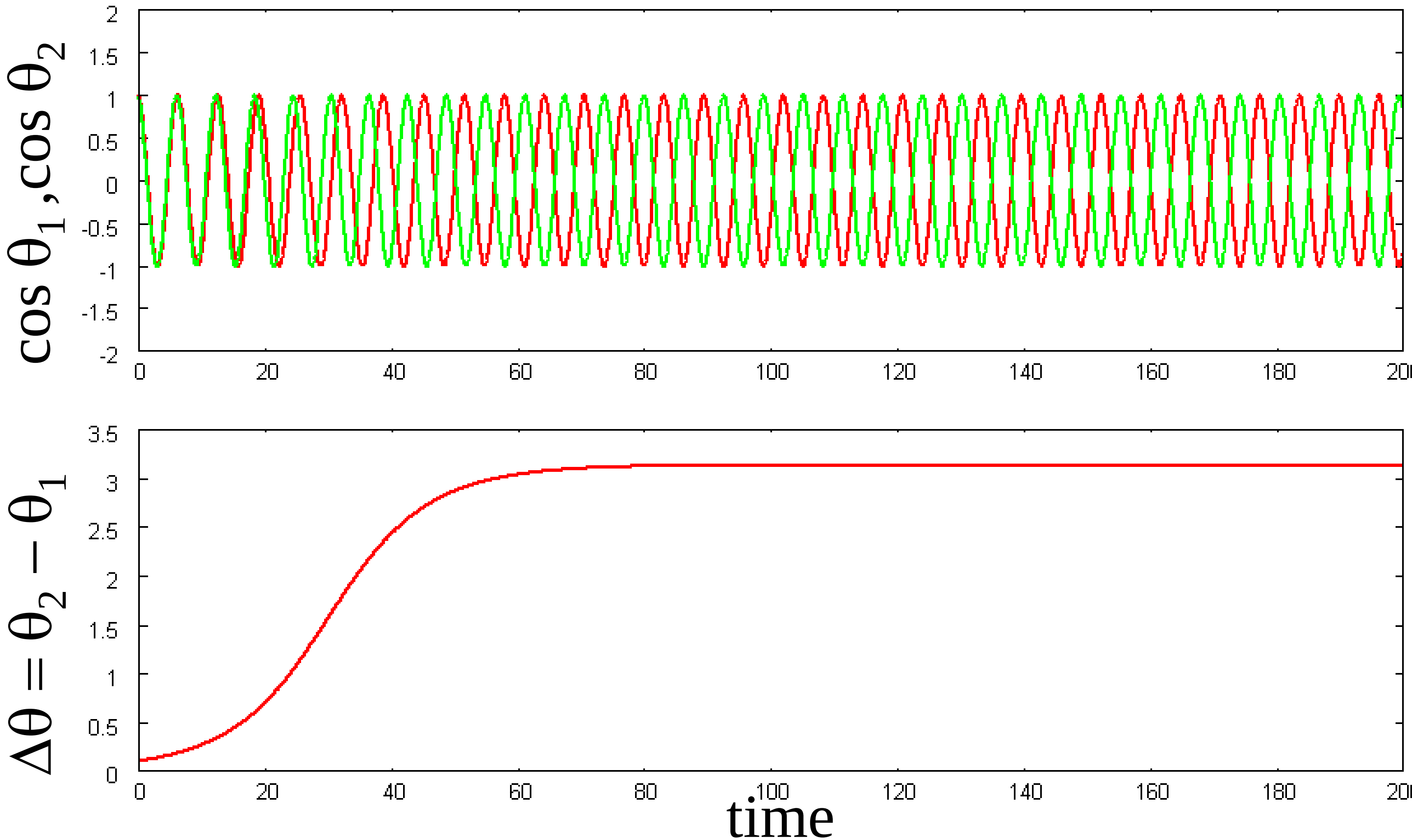


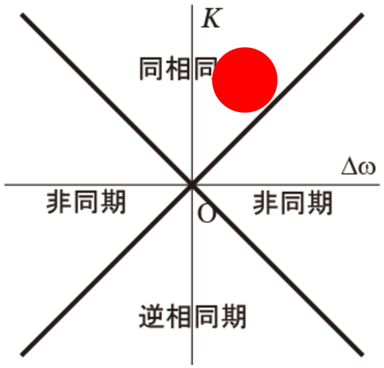


$$\begin{cases} \dfrac{\mathrm{d}\theta_1}{\mathrm{d}t} = \omega_1 + K\sin(\theta_2 - \theta_1) \\ \dfrac{\mathrm{d}\theta_2}{\mathrm{d}t} = \omega_2 + K\sin(\theta_1 - \theta_2) \end{cases}$$

$\omega_1 = \omega_2 = 1$

$K = -0.05$

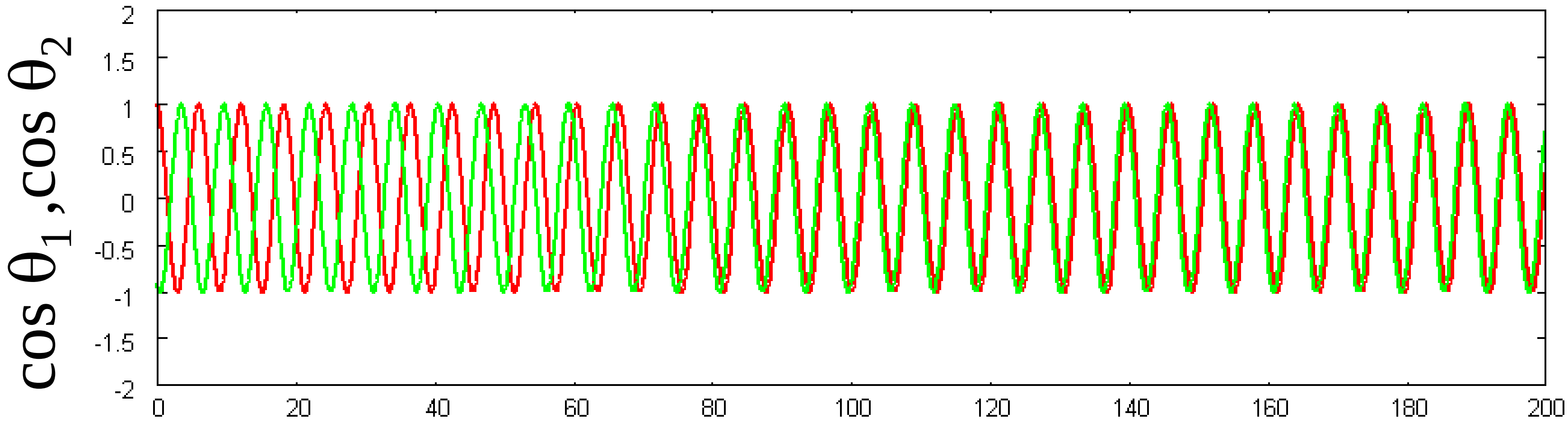


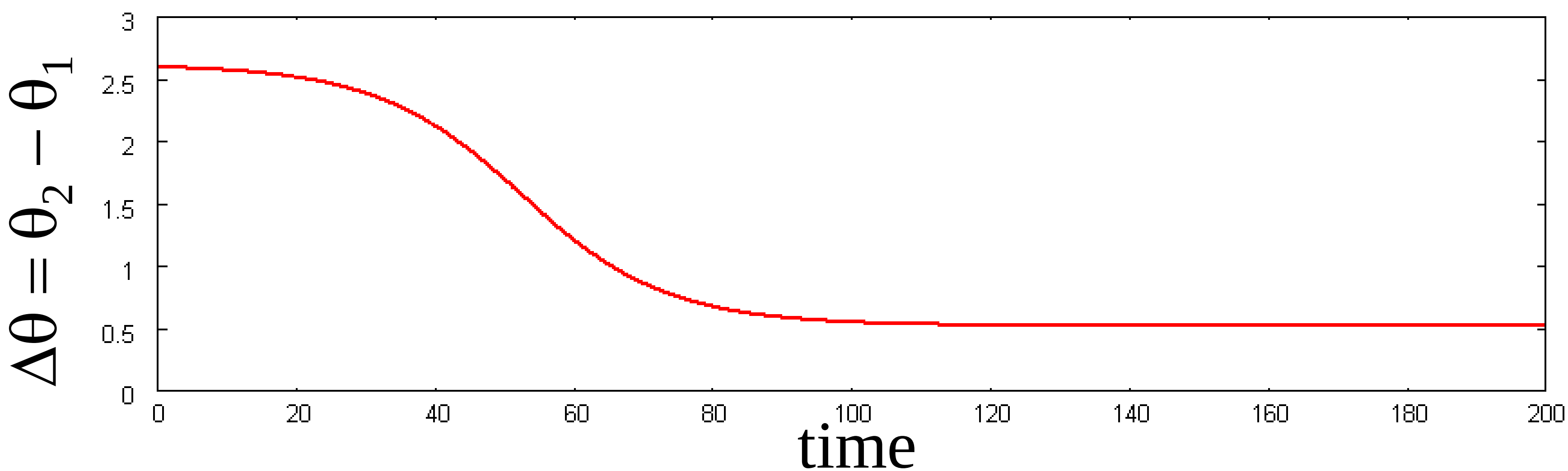


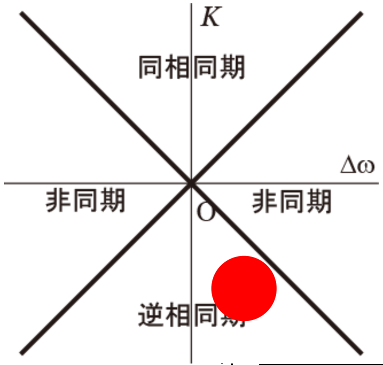
$$
\begin{cases}
\dfrac{\mathrm{d}\theta_1}{\mathrm{d}t} = \omega_1 + K\sin(\theta_2 - \theta_1) \\
\dfrac{\mathrm{d}\theta_2}{\mathrm{d}t} = \omega_2 + K\sin(\theta_1 - \theta_2)
\end{cases}
$$

$\omega_1 = 1, \ \ \omega_2 = 1.05$

$K = 0.05$



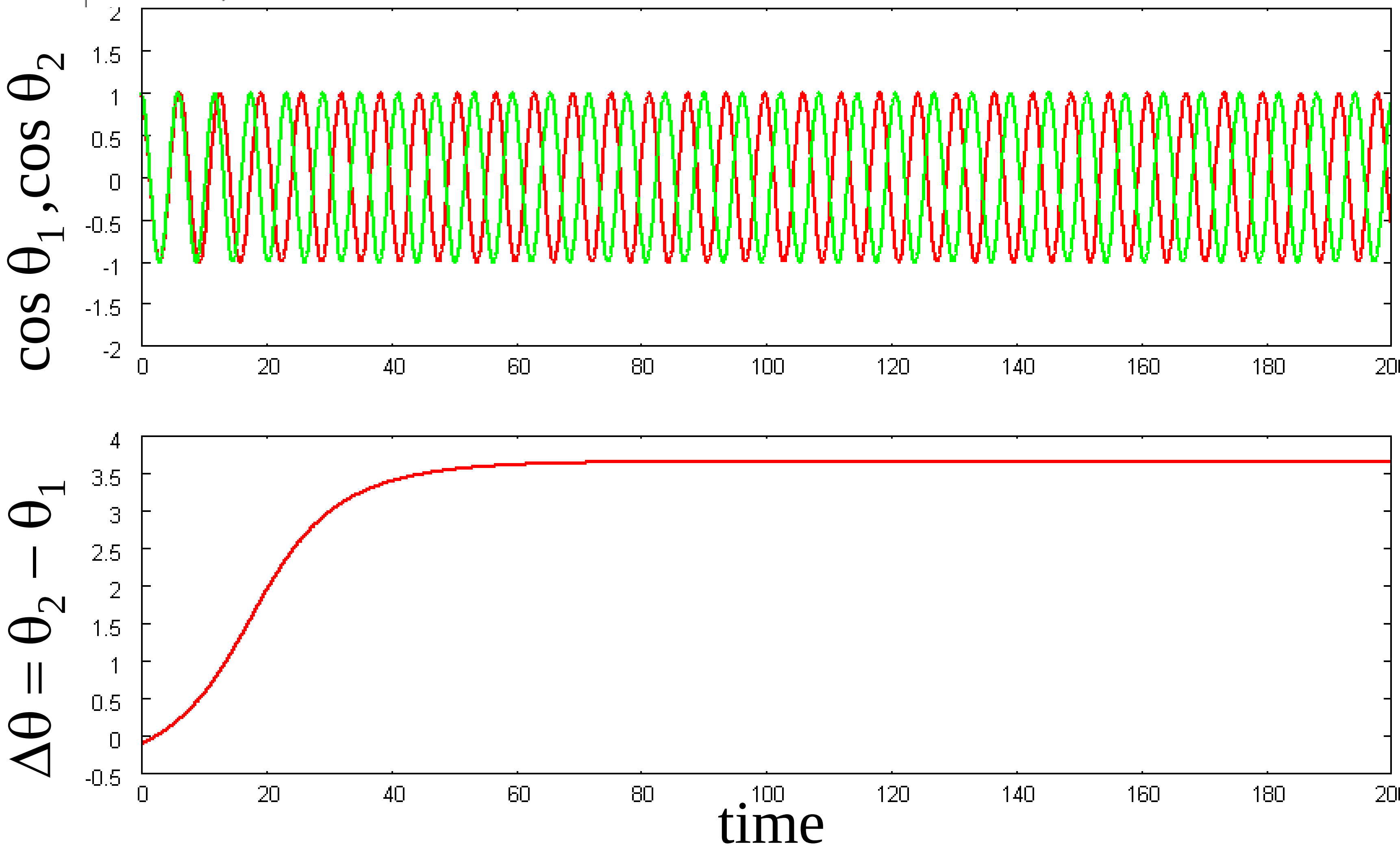


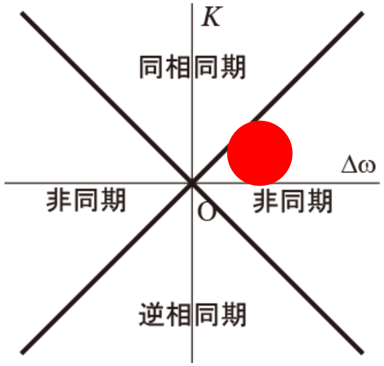


$$
\begin{cases}
\dfrac{\mathrm{d}\theta_1}{\mathrm{d}t} = \omega_1 + K\sin(\theta_2 - \theta_1) \\
\dfrac{\mathrm{d}\theta_2}{\mathrm{d}t} = \omega_2 + K\sin(\theta_1 - \theta_2)
\end{cases}
$$

$\omega_1 = 1,\ \omega_2 = 1.05$

$K = -0.05$

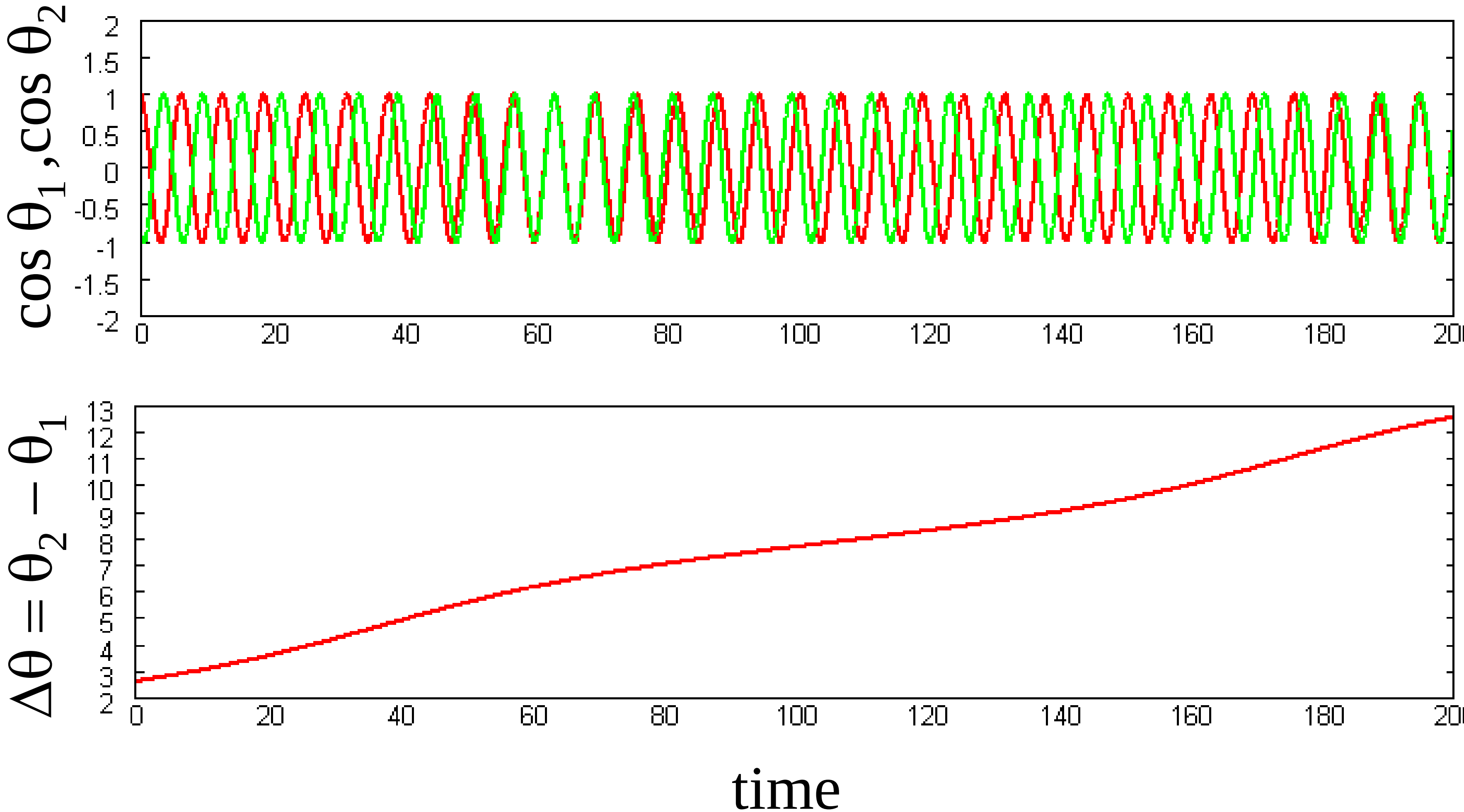




$$
\begin{cases}
\dfrac{d\theta_1}{dt} = \omega_1 + K\sin(\theta_2 - \theta_1) \\
\dfrac{d\theta_2}{dt} = \omega_2 + K\sin(\theta_1 - \theta_2)
\end{cases}
$$

$\omega_1 = 1,\ \omega_2 = 1.05$

$K = 0.01$



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物性物理学C

反応拡散系とパターン形成

Turingパターン（静止パターン）

$$\frac{\partial u}{\partial t} = -u^3 + u - 4v + D_u \nabla^2 u$$

$$\frac{\partial v}{\partial t} = u - 3v - a + D_v \nabla^2 v$$

$a = 0$では$u = v = 0$が固定点

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} \Delta u \\ \Delta v \end{pmatrix}$$

$\mathrm{tr}\,A = -2$、$\det A = 1$より安定

$$u = 0 + \int \mathrm{d}k\ \Delta u(k) e^{ikx}$$

$$v = 0 + \int \mathrm{d}k\ \Delta v(k) e^{ikx}$$ とおいて

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\Delta u \\ \Delta v\end{pmatrix} = \begin{pmatrix}1 - D_u k^2 & -4 \\ 1 & -3 - D_u k^2\end{pmatrix}\begin{pmatrix}\Delta u \\ \Delta v\end{pmatrix}$$

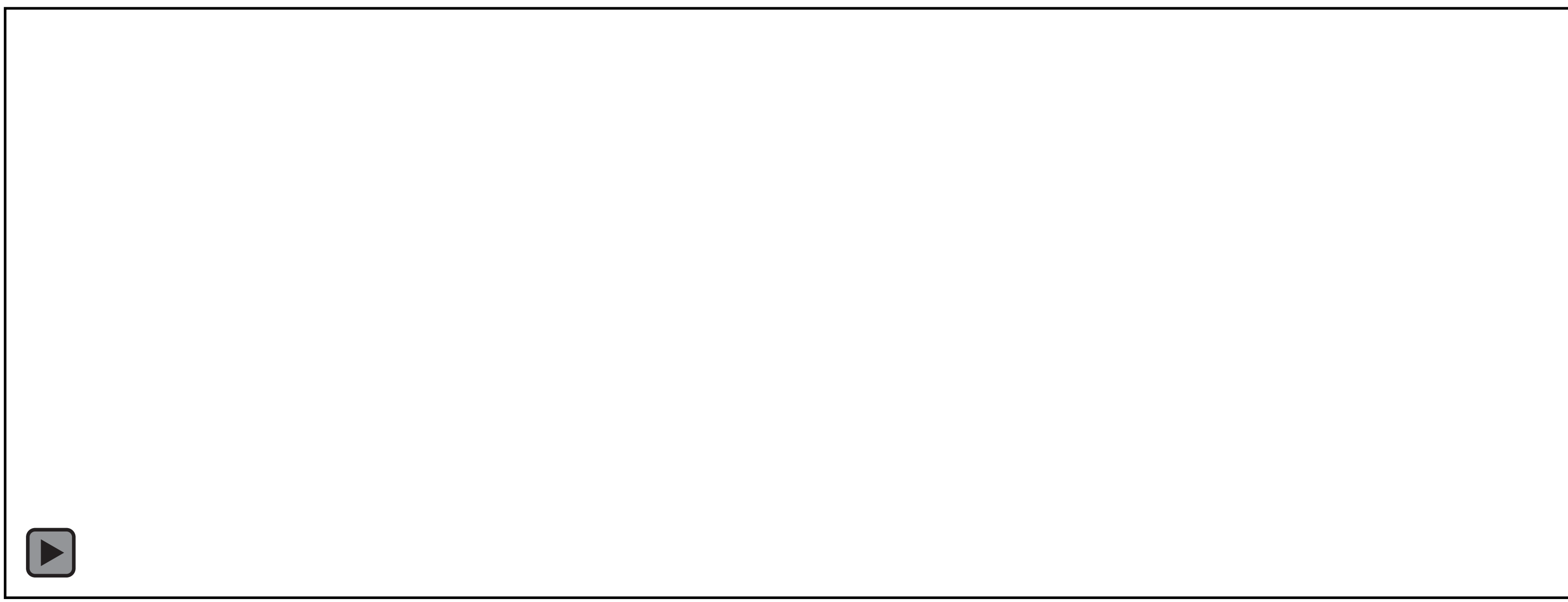
不安定化する条件は

$$D_v > 9D_u$$

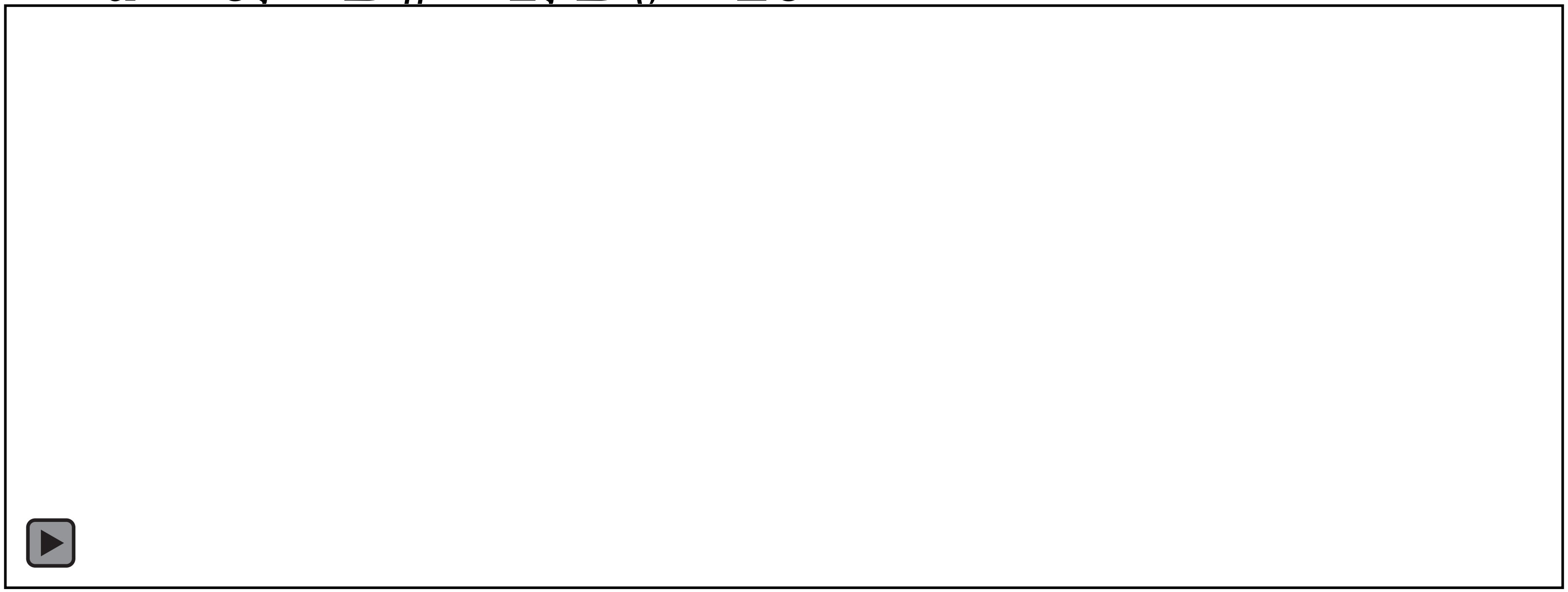
はじめに不安定化する波数は $k = \sqrt{\frac{1}{2D_u} - \frac{3}{2D_v}}$

1次元では

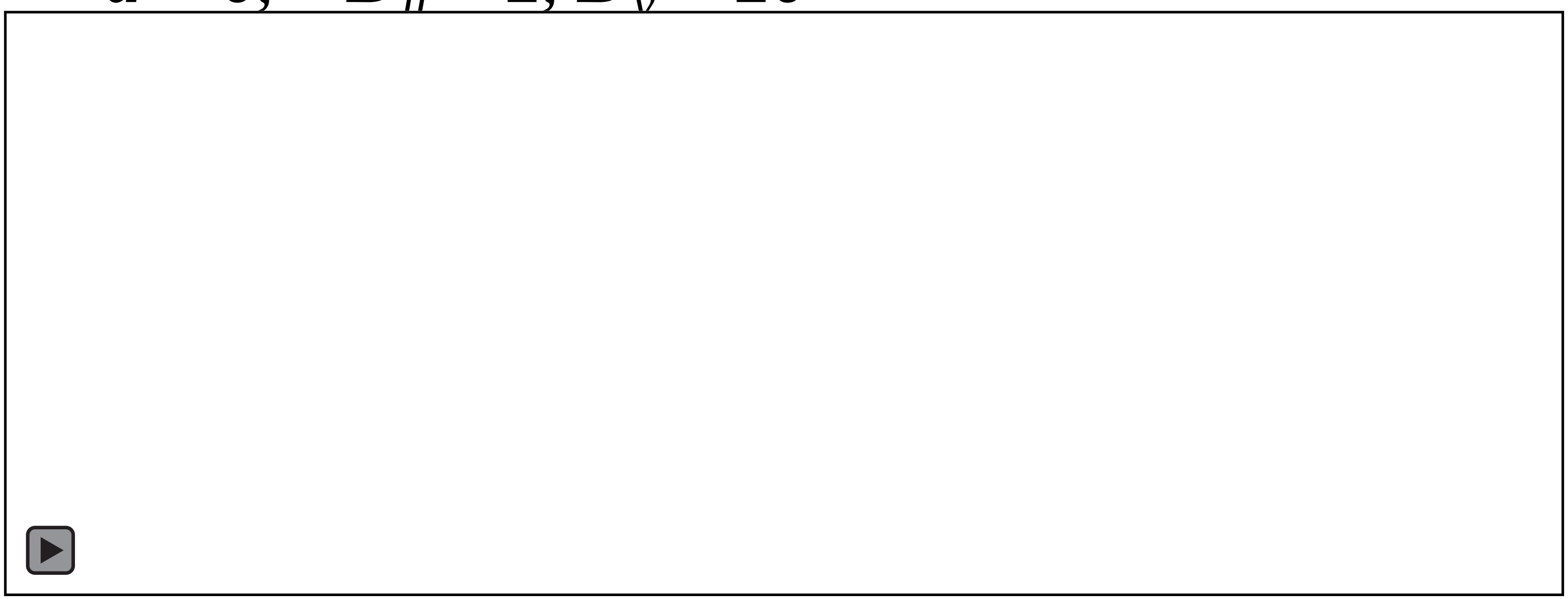
$$a = 0, \quad D_u = 1, D_v = 5$$



$a = 0, \quad D_u = 1, D_v = 20$

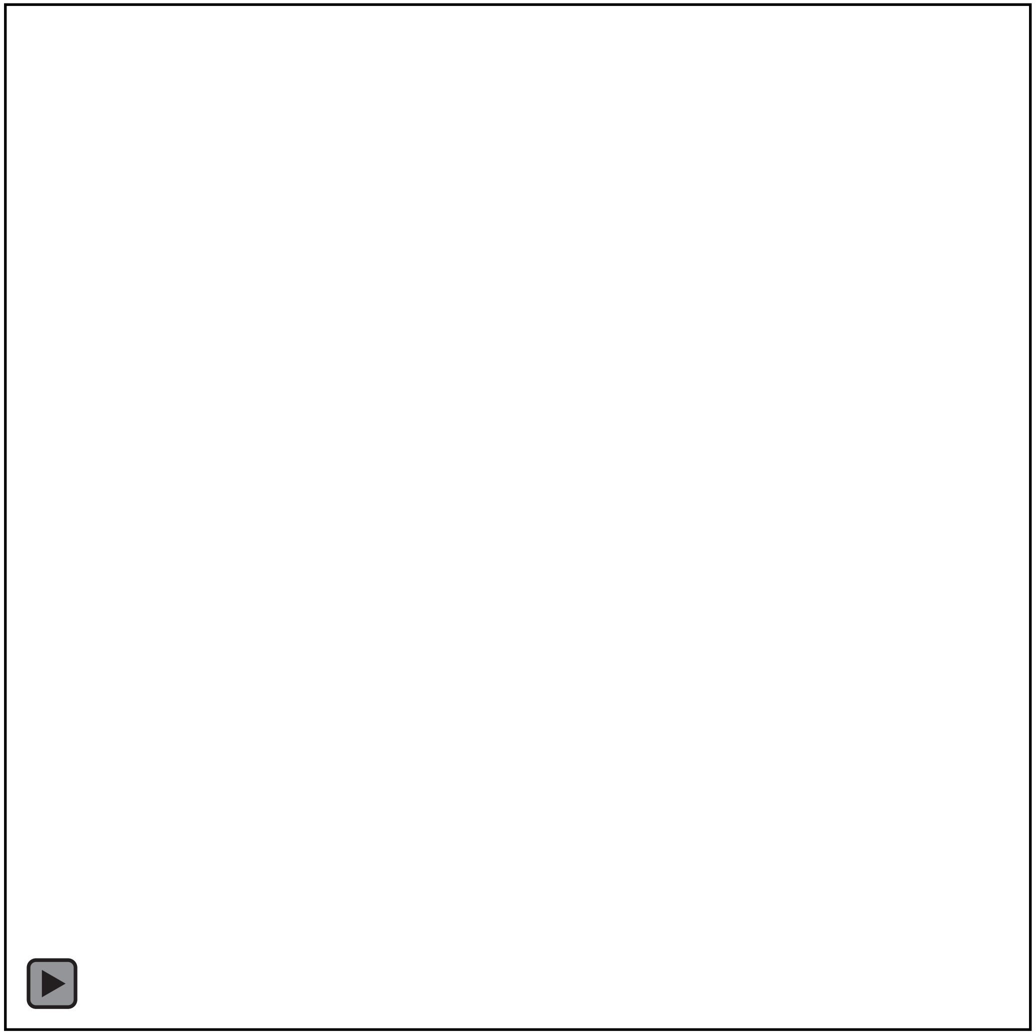


$a = 0, \quad D_u = 2, D_v = 20$

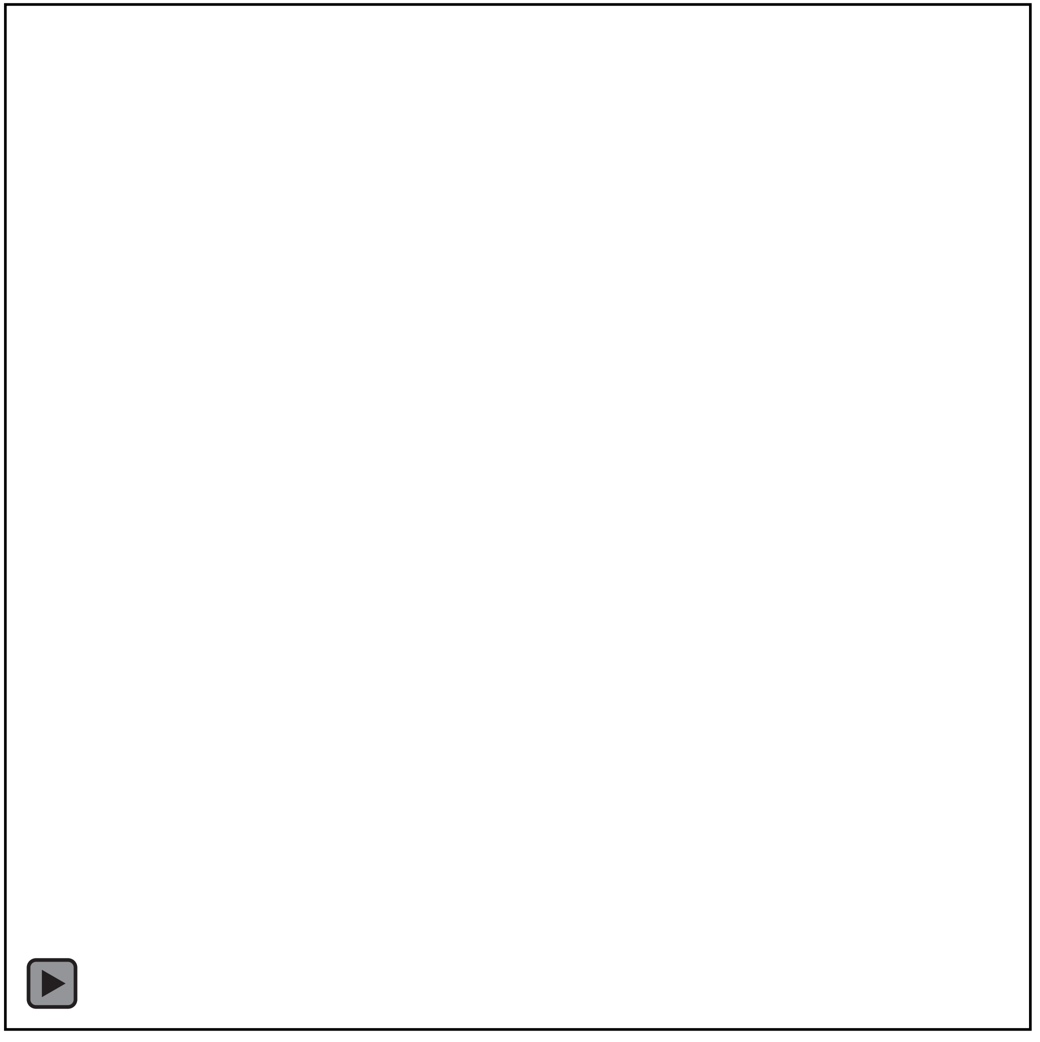


2次元では

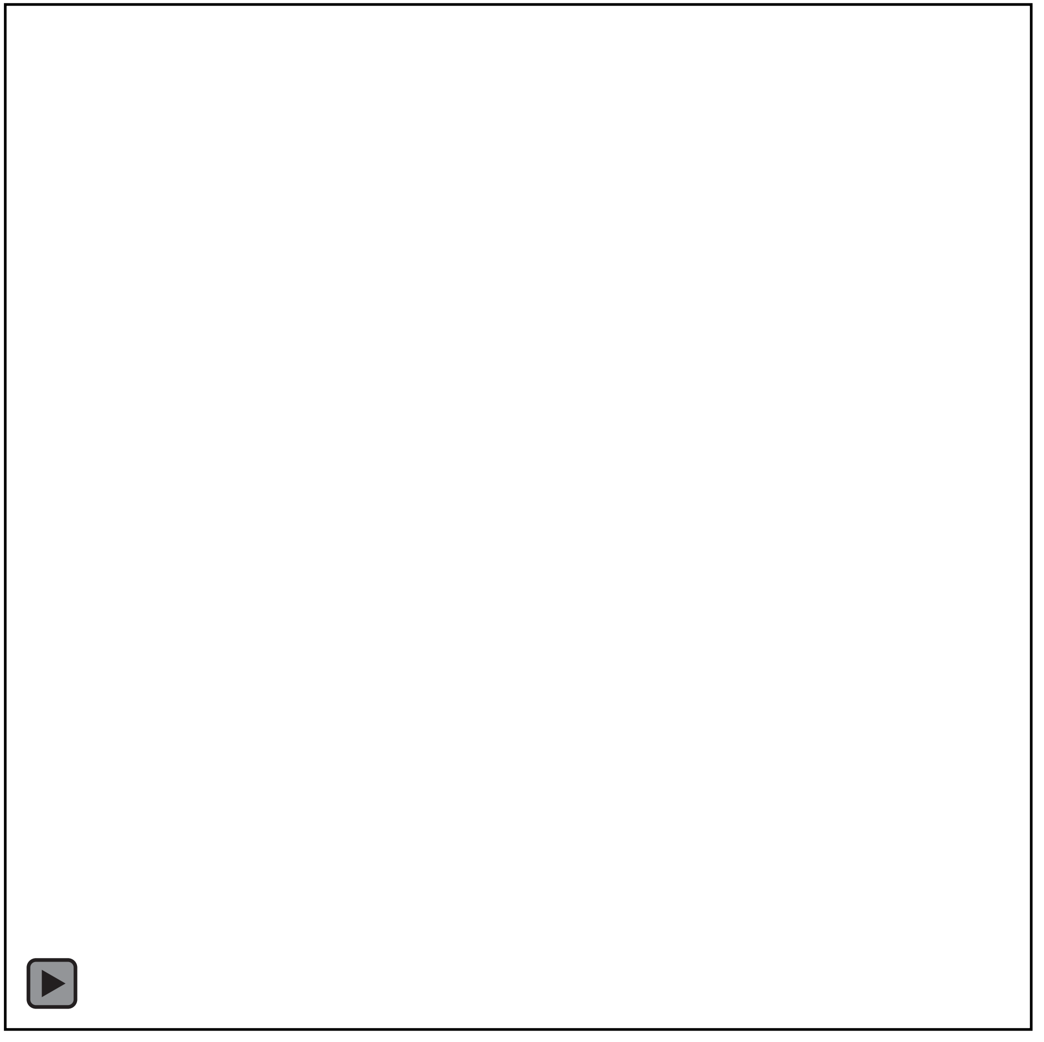
$a = 0$ $D_u = 1, D_v = 5$



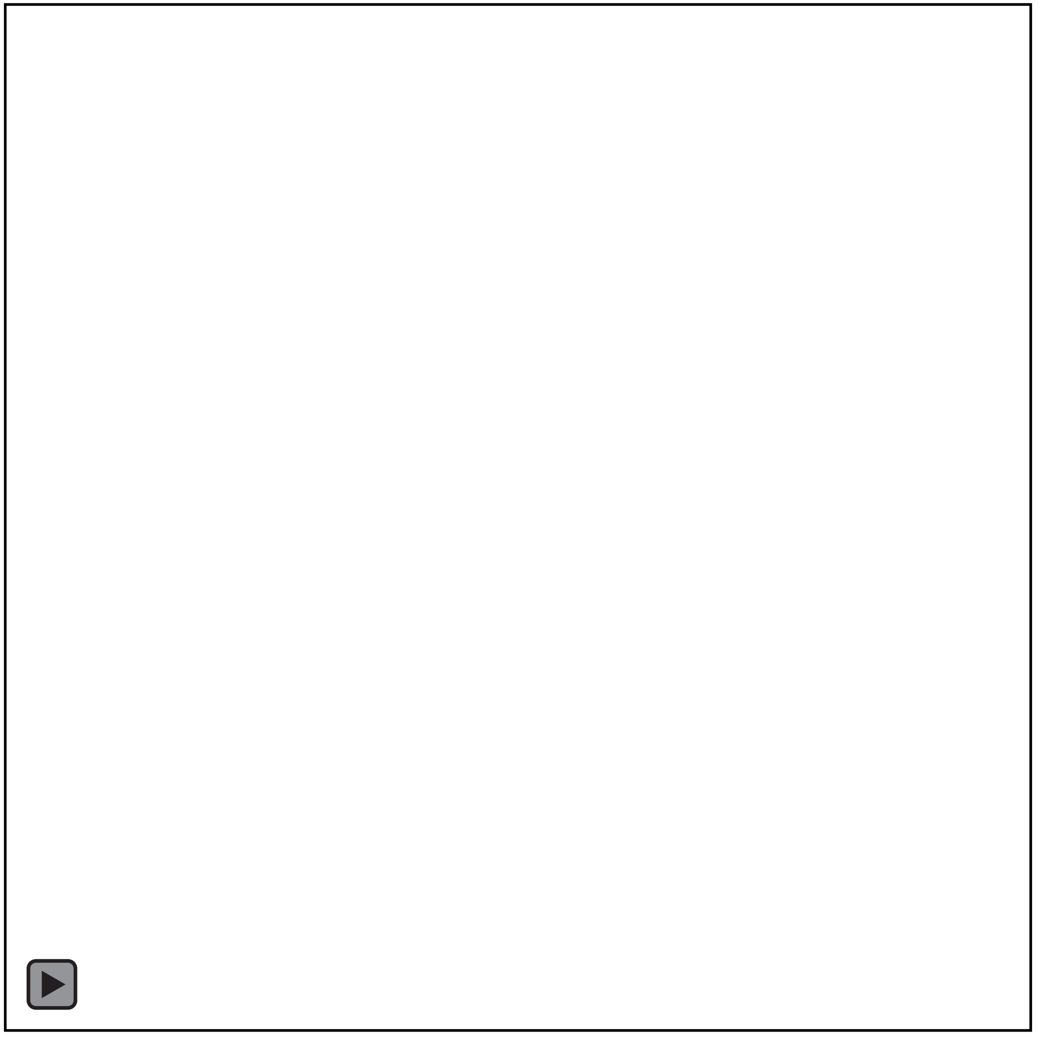
$a = 0$ $D_u = 1, D_v = 20$



$a = 0$ $D_u = 2, D_v = 20$

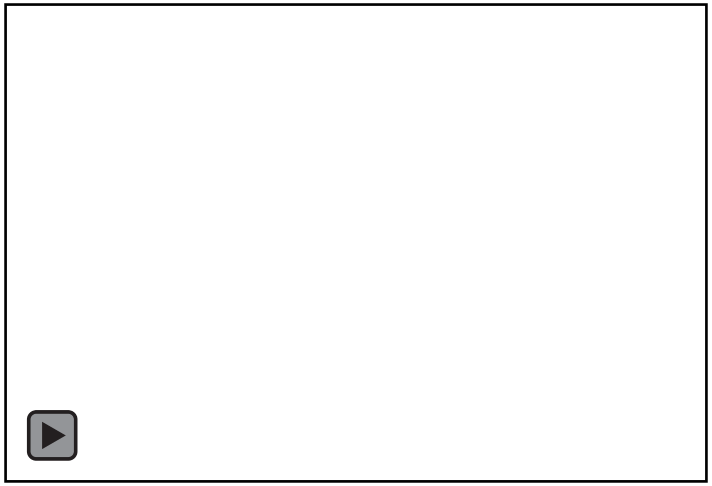


$a = 0.05$ $D_u = 1, D_v = 20$



Belousov-Zhabotinsky反応

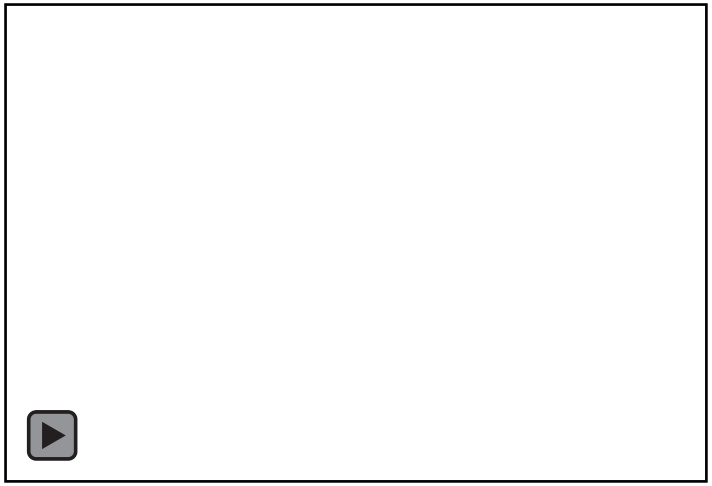
a) Target Pattern



（4倍速）

3 mm

b) Spiral Pattern



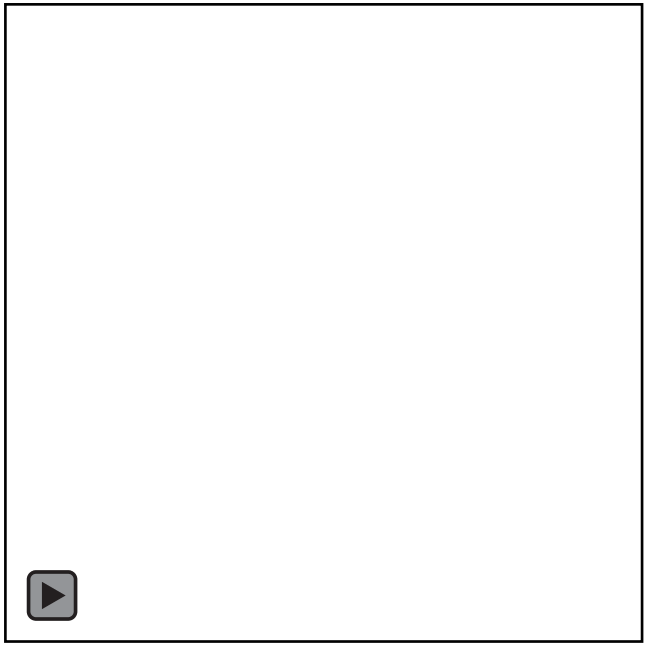
（4倍速）

3 mm

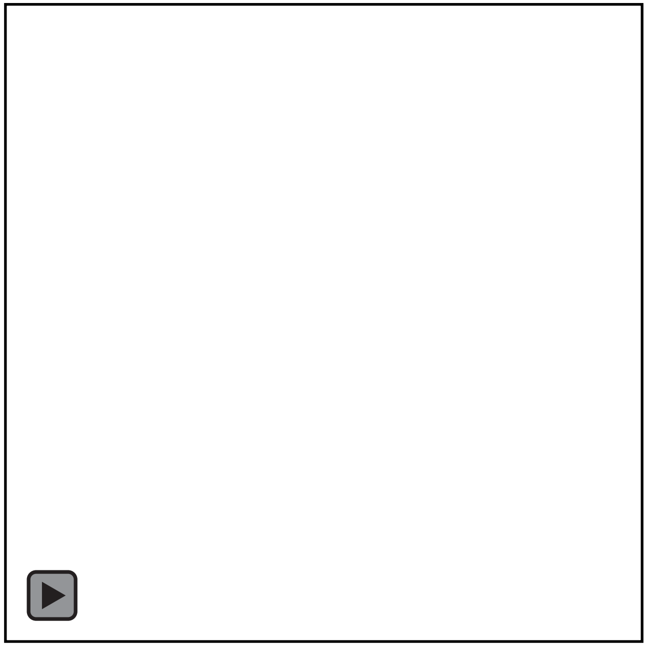
$$\frac{\partial U}{\partial t} = \frac{1}{\varepsilon}\left(U(1-U) - fV\frac{U-q}{U+q} \right) + D_U \nabla^2 U$$

$$\frac{\partial V}{\partial t} = U - V + D_V \nabla^2 V$$

Target Pattern



Spiral Pattern



（Keener-Tyson version Oregonatorを使用）