

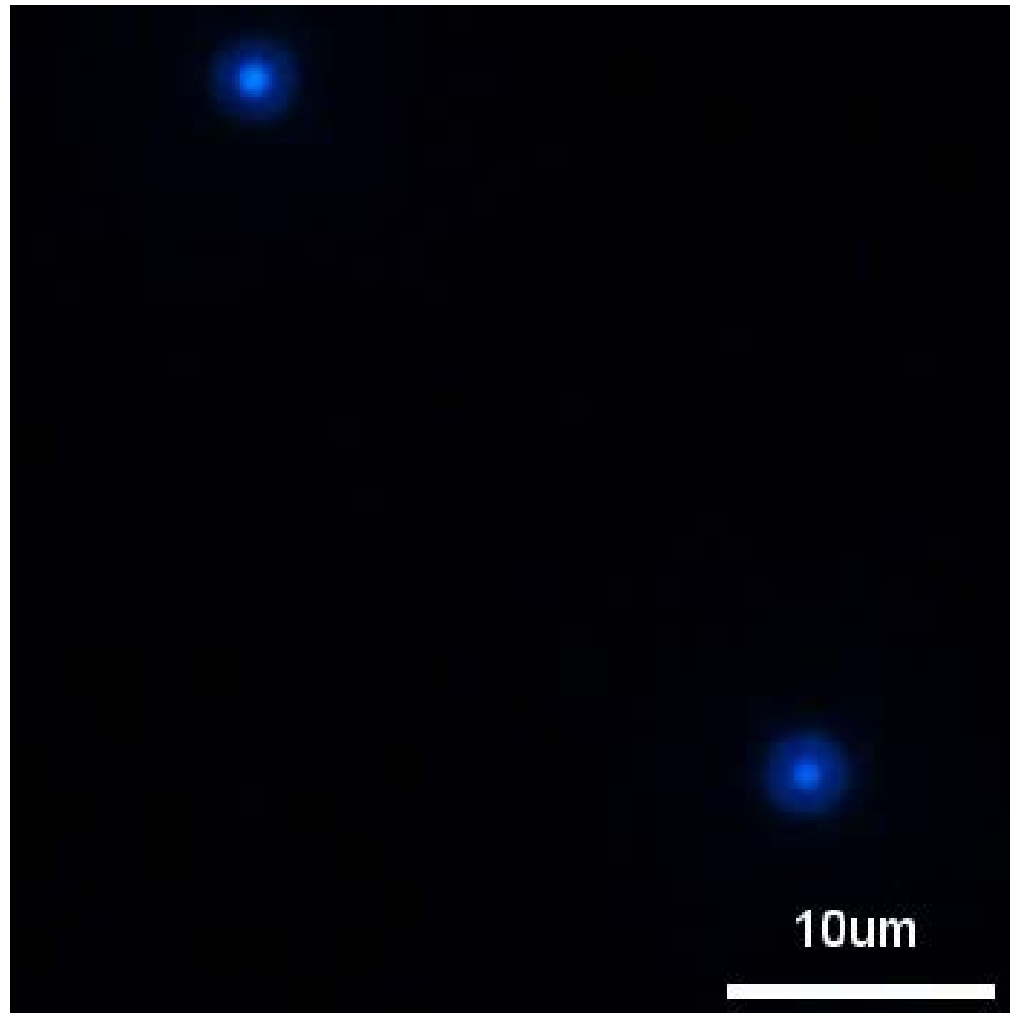
2014.10.21
物性物理学C

ランダムウォークと拡散方程式

北 畑 裕 之

ブラウン運動

ナノ粒子



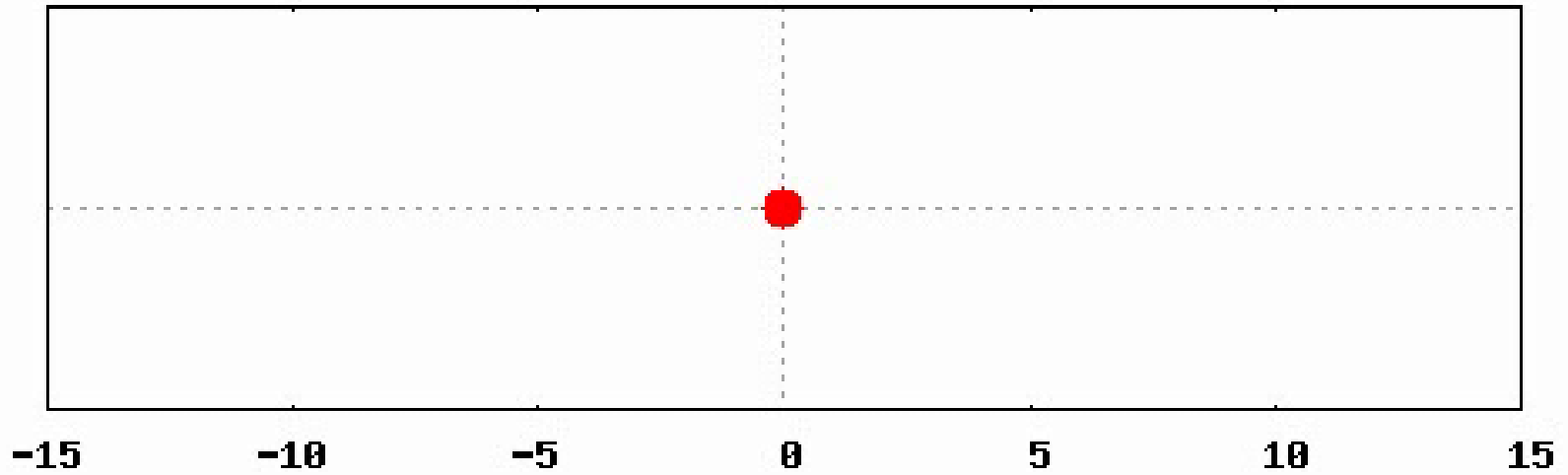
ランダムウォーク

$$x_{i+1} = x_i + \xi_i$$

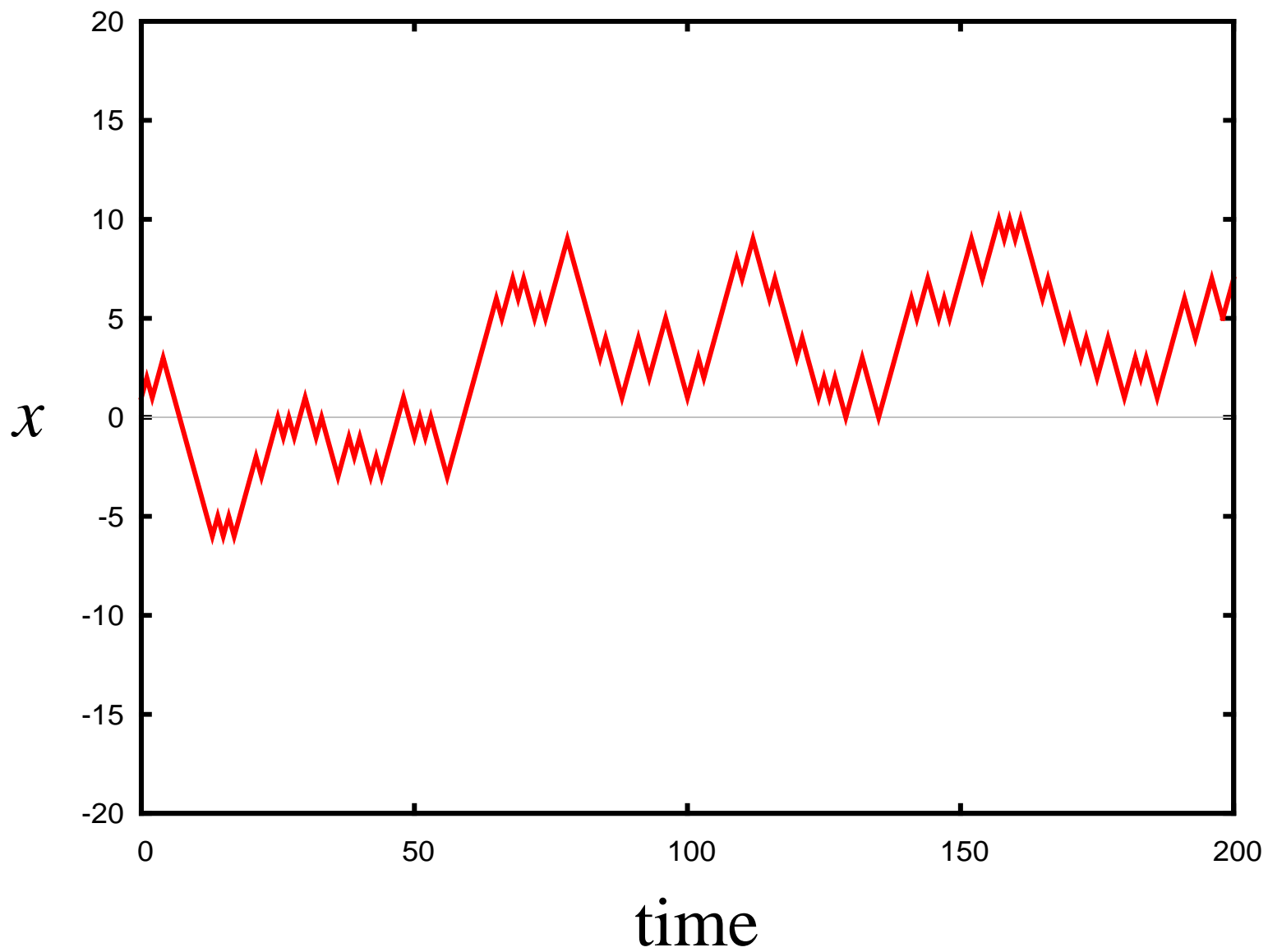
$$\xi_i = \begin{cases} 1 & (\text{Prob. } 1/2) \\ -1 & (\text{Prob. } 1/2) \end{cases}$$

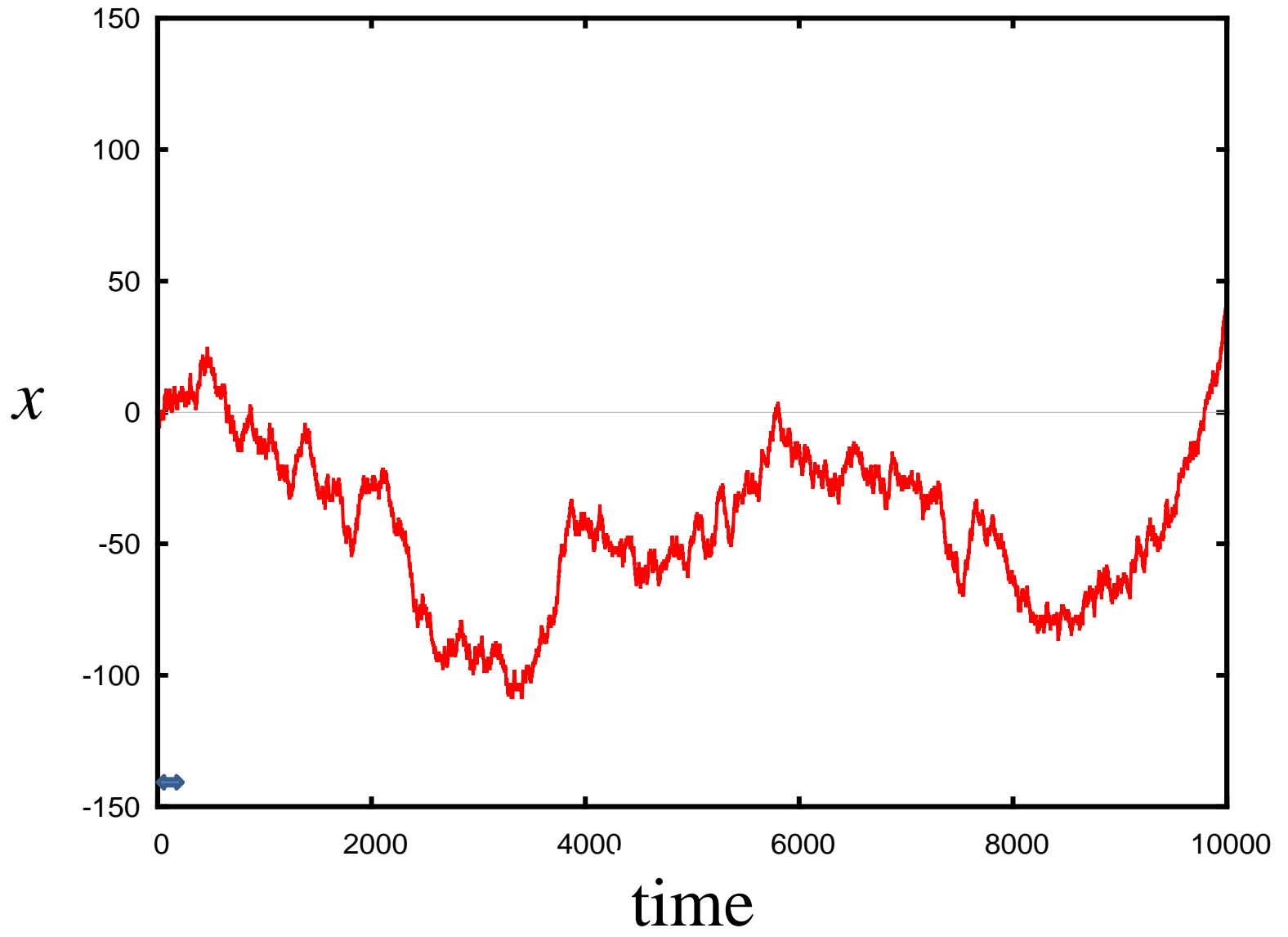
$$\langle \xi_i \xi_j \rangle = \delta_{ij}$$

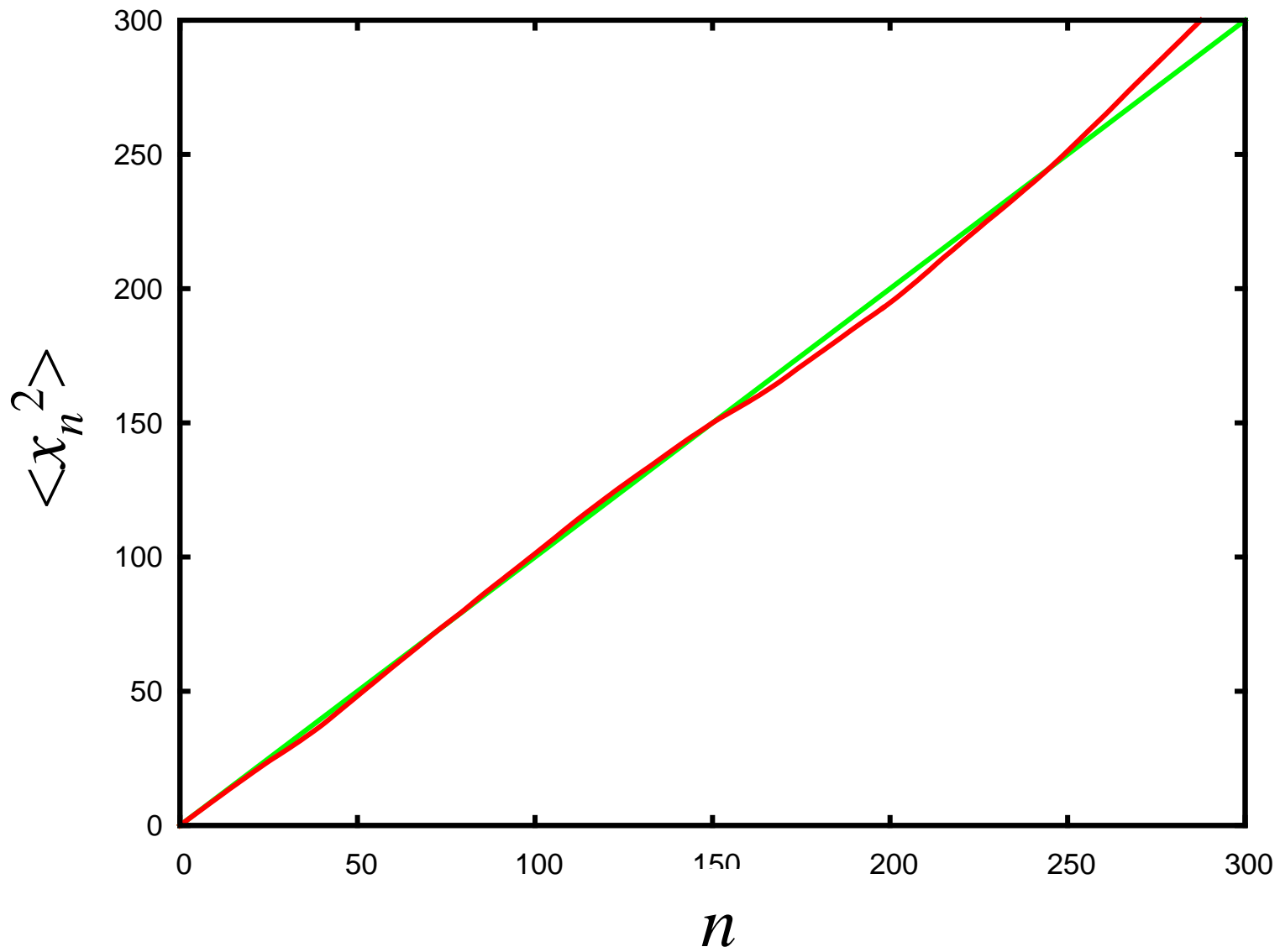
1次元ランダムウォーク



x







$$\begin{aligned}
\langle x_n^2 \rangle &= \left\langle \left[\sum_{k=0}^{n-1} \xi_k \right]^2 \right\rangle \\
&= \left\langle \sum_{k=0}^{n-1} \sum_{k'=0}^{n-1} \xi_k \xi_{k'} \right\rangle \\
&= \sum_{k=0}^{n-1} \sum_{k'=0}^{n-1} \langle \xi_k \xi_{k'} \rangle \\
&= \sum_{k=0}^{n-1} \sum_{k'=0}^{n-1} \delta_{kk'} \\
&= n
\end{aligned}$$

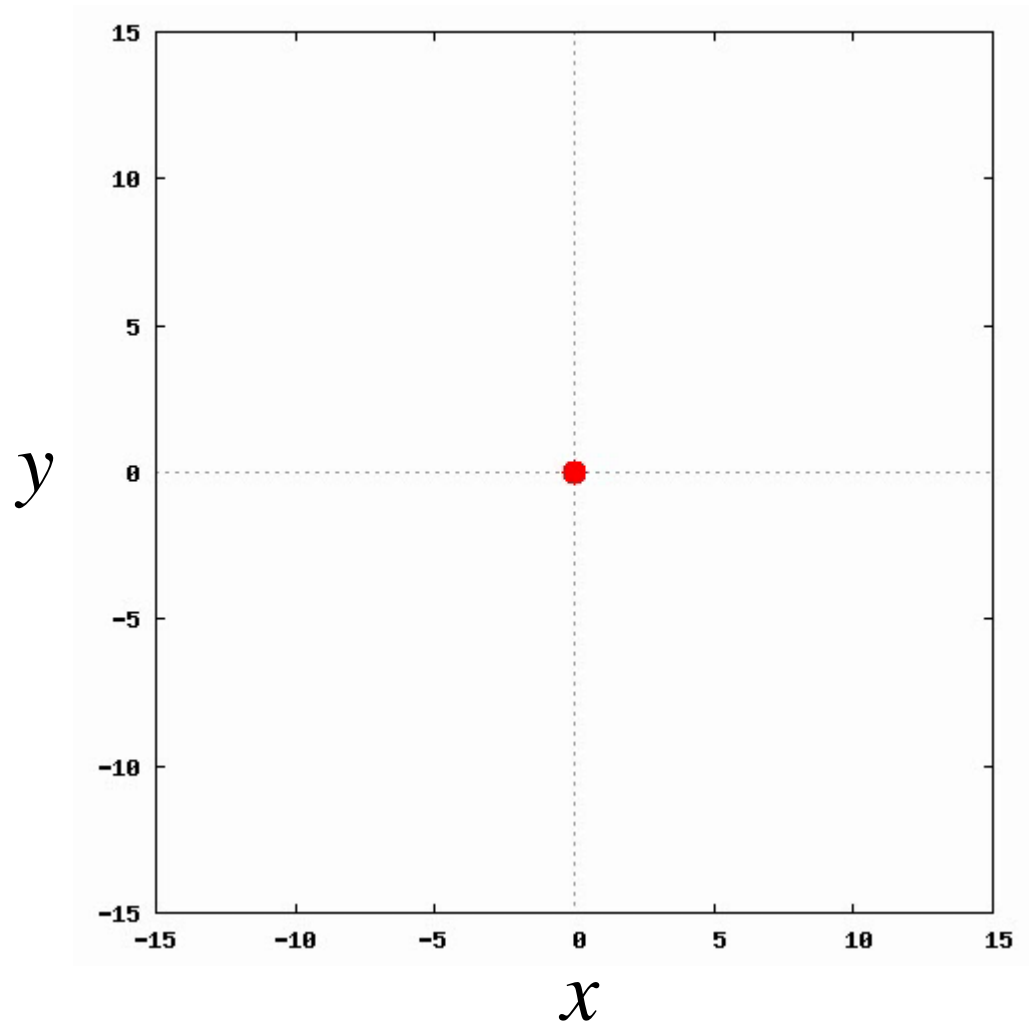
2次元の場合

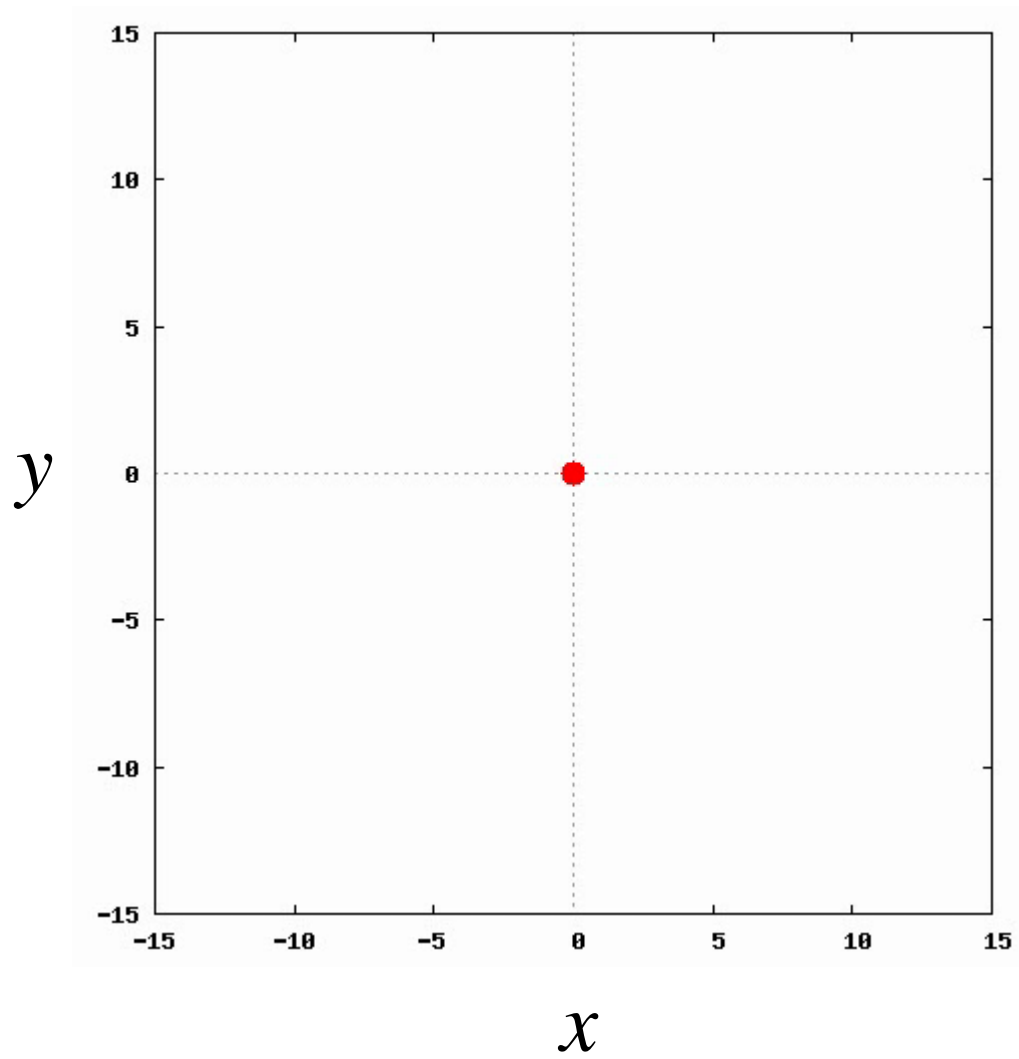
$$\mathbf{r}_{i+1} = \mathbf{r}_i + \boldsymbol{\xi}_i \quad \mathbf{r}_i = (x_i, y_i)$$

$$\boldsymbol{\xi}_i = \begin{cases} (1, 0) & (\text{Prob. } 1/4) \\ (-1, 0) & (\text{Prob. } 1/4) \\ (0, 1) & (\text{Prob. } 1/4) \\ (0, -1) & (\text{Prob. } 1/4) \end{cases}$$

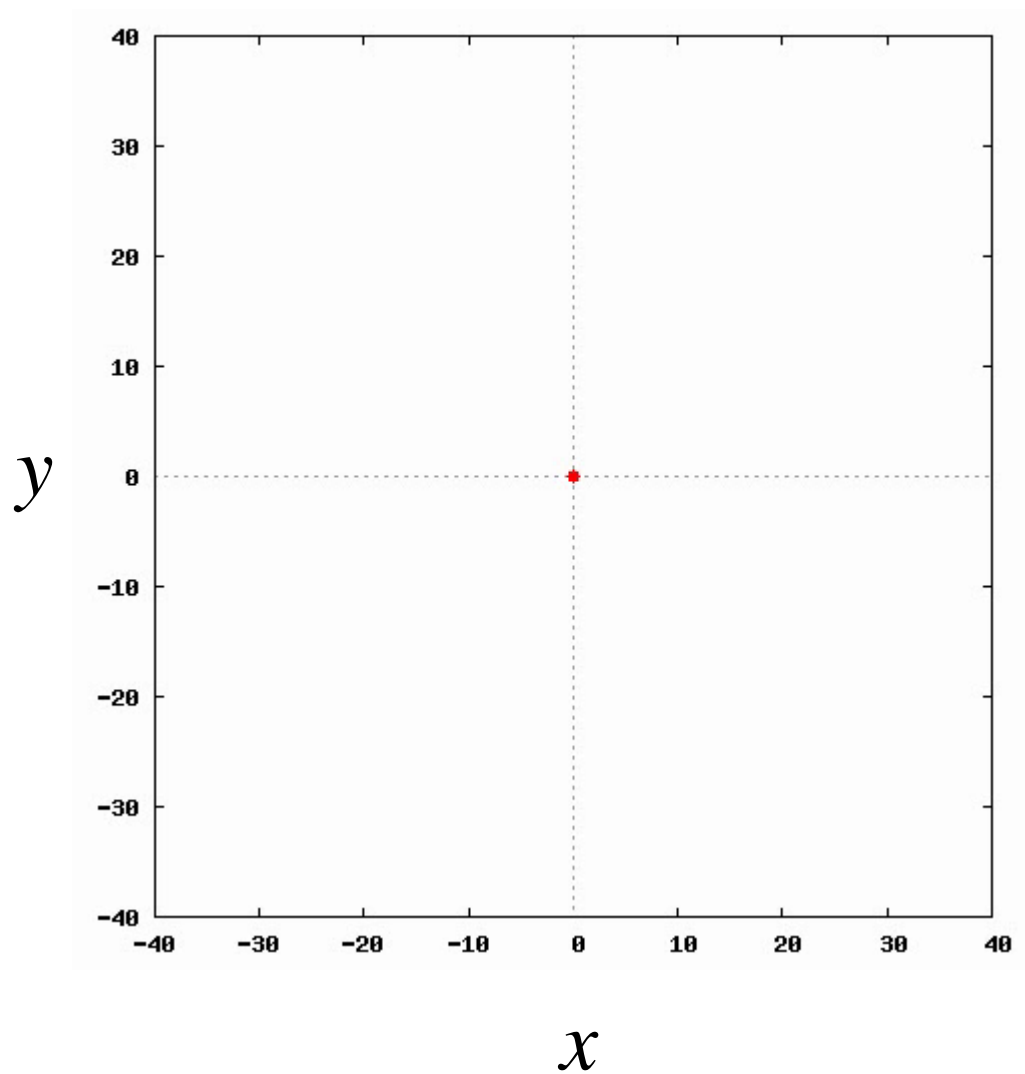
$$\langle \boldsymbol{\xi}_i \cdot \boldsymbol{\xi}_j \rangle = \delta_{ij}$$

2次元ランダムウォーク





100個の粒子



1000個の粒子

