

物性物理学C

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分岐理論

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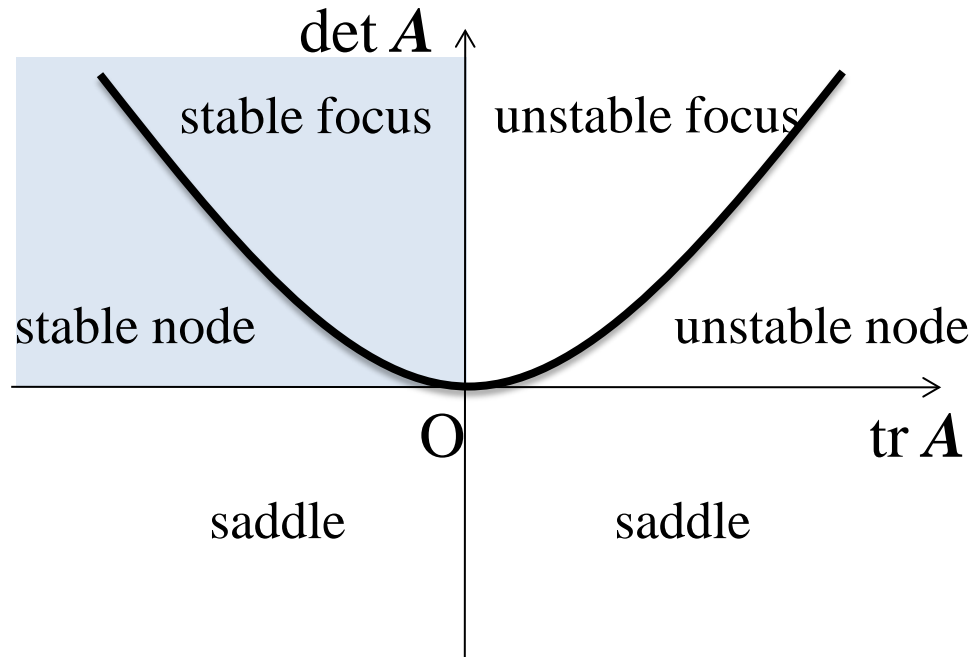
おもな分岐の種類

- サドル・ノード分岐
- ピッチフォーク分岐
- トランスクリティカル分岐 (安定性交替分岐)
- ホップ分岐

分岐とは?

2変数の力学系で考える

$$\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix} \quad \mathbf{A}: \text{関数行列}$$



ただし、ホップ分岐以外では、 x 方向と y 方向は独立しているので、 $dy/dt = a y$ として考えることにする。

- サドル・ノード分岐

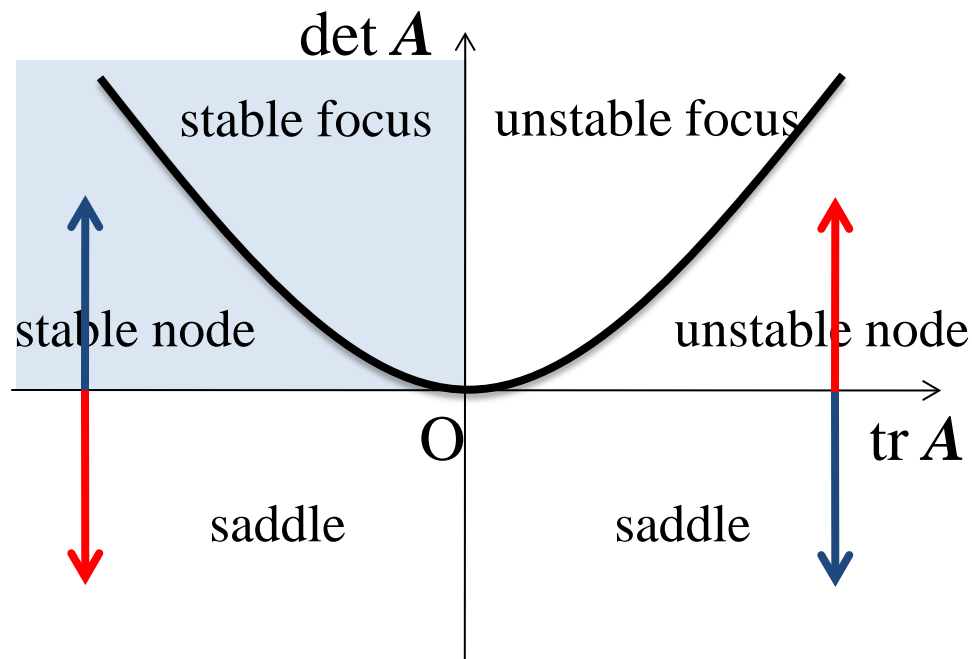
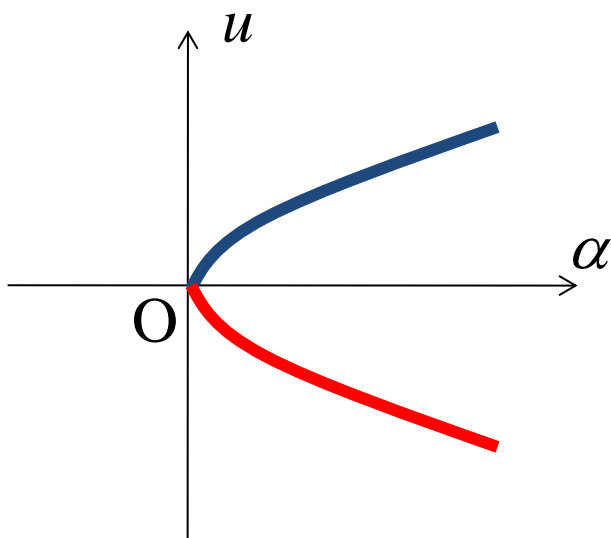
$$u = \pm \sqrt{\alpha}, \quad v = 0 \quad (\alpha \geq 0) \text{ まわりで}$$

$$\begin{aligned} \frac{du}{dt} &= \alpha - u^2 \\ \frac{dv}{dt} &= kv \end{aligned}$$

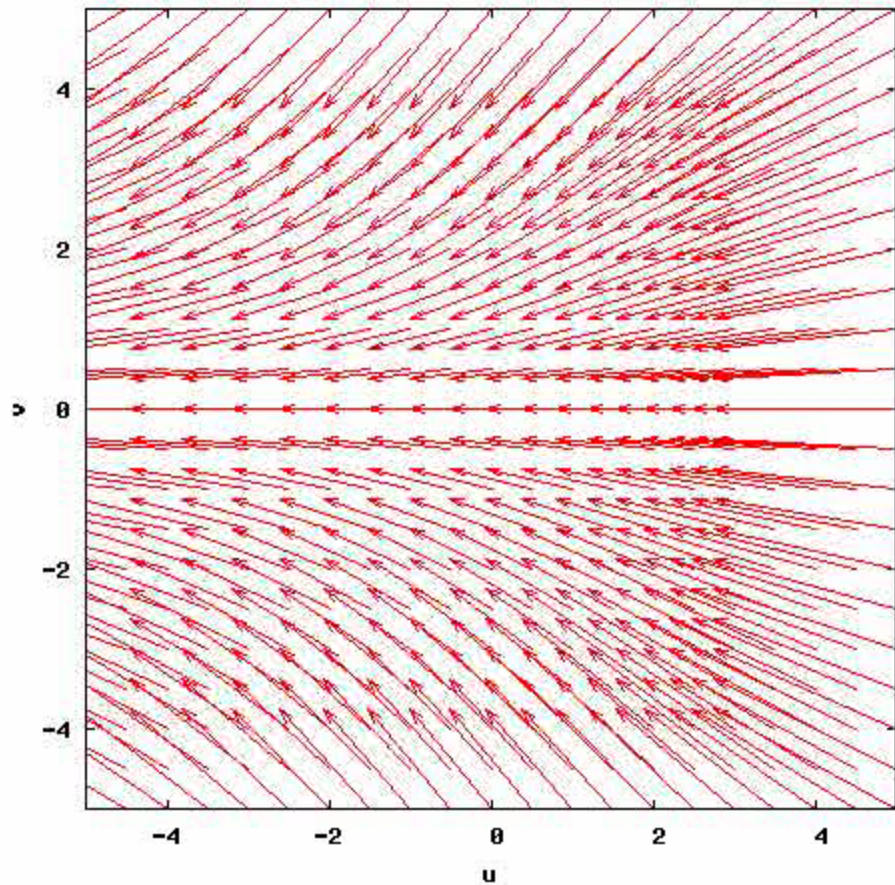
固定点:

$$\begin{aligned} u &= \pm \sqrt{\alpha} \\ v &= 0 \end{aligned}$$

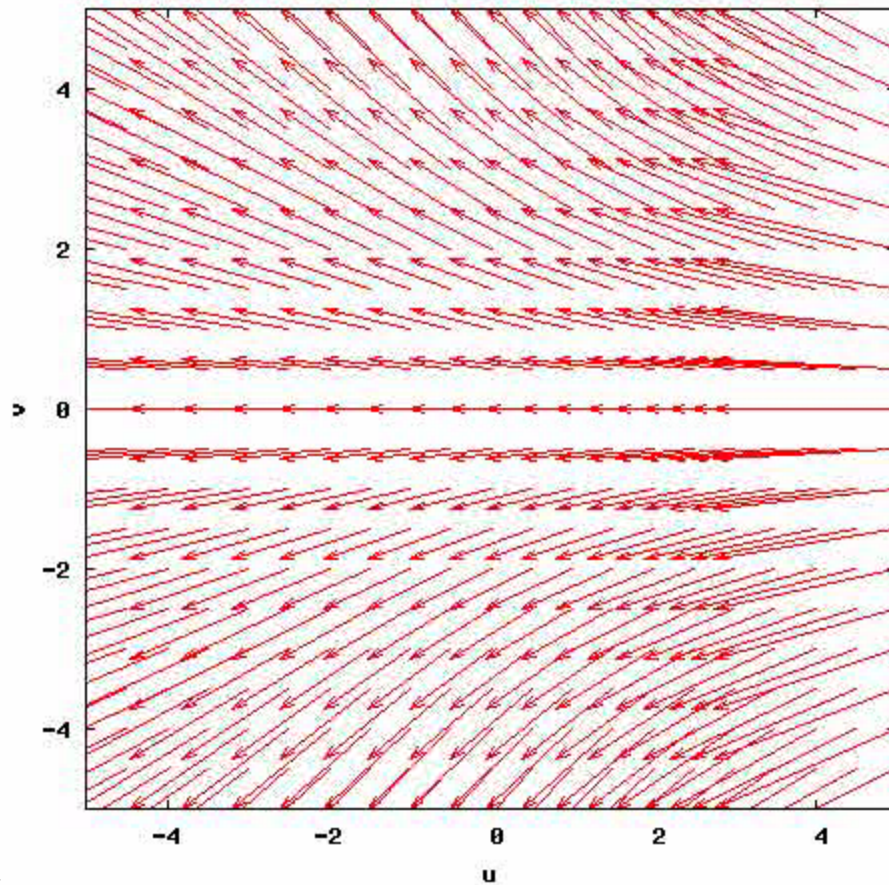
$$\mathbf{A} = \begin{pmatrix} \mp 2\sqrt{\alpha} & 0 \\ 0 & k \end{pmatrix}$$



alpha = -20.00

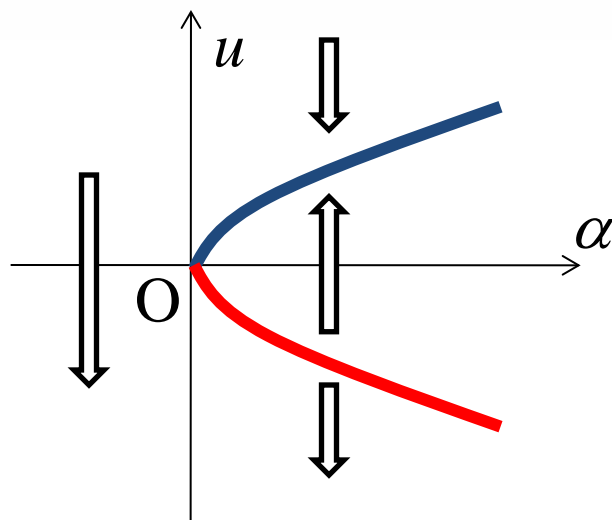


alpha = -20.00



$$\frac{du}{dt} = \alpha - u^2$$
$$\frac{dv}{dt} = kv$$

$$k = -5$$



$$k = 5$$

- ピッチフォーク分岐

$$u = \pm \sqrt{\alpha}, \quad v = 0 \quad (\alpha \geq 0) \text{ まわりで}$$

$$\frac{du}{dt} = \alpha u - u^3$$

$$\mathbf{A} = \begin{pmatrix} -2\alpha & 0 \\ 0 & k \end{pmatrix}$$

$$\frac{dv}{dt} = kv$$

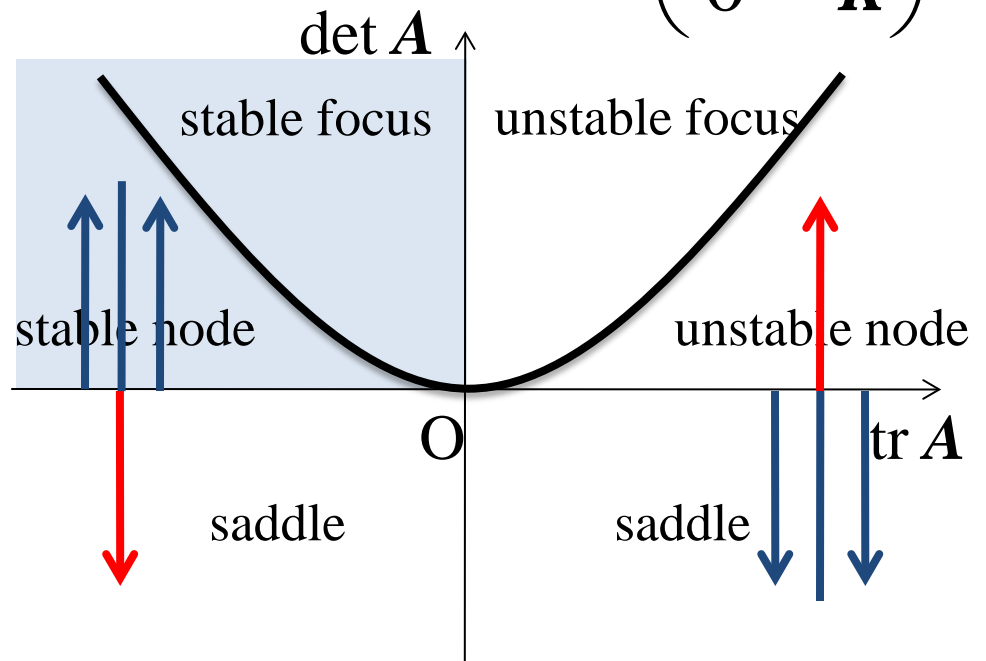
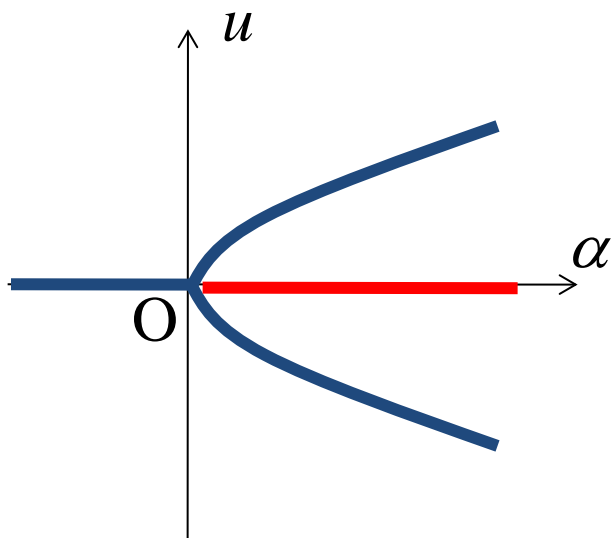
固定点:

$$u = 0, \pm \sqrt{\alpha}$$

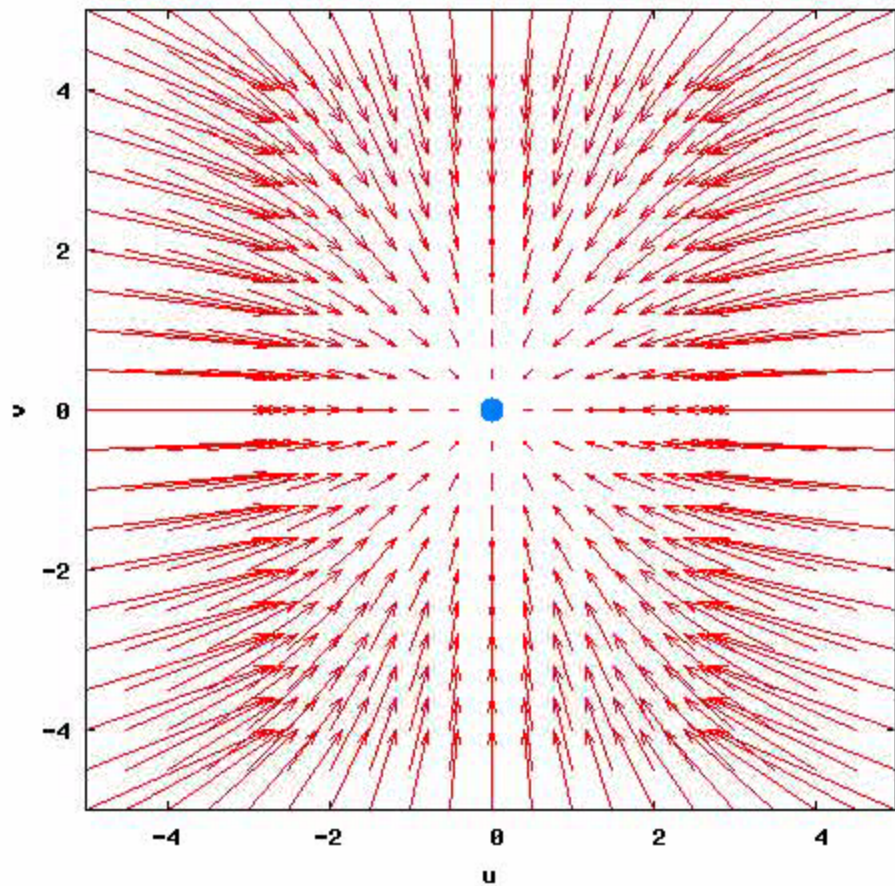
$$u = 0, \quad v = 0 \text{ まわりで}$$

$$v = 0$$

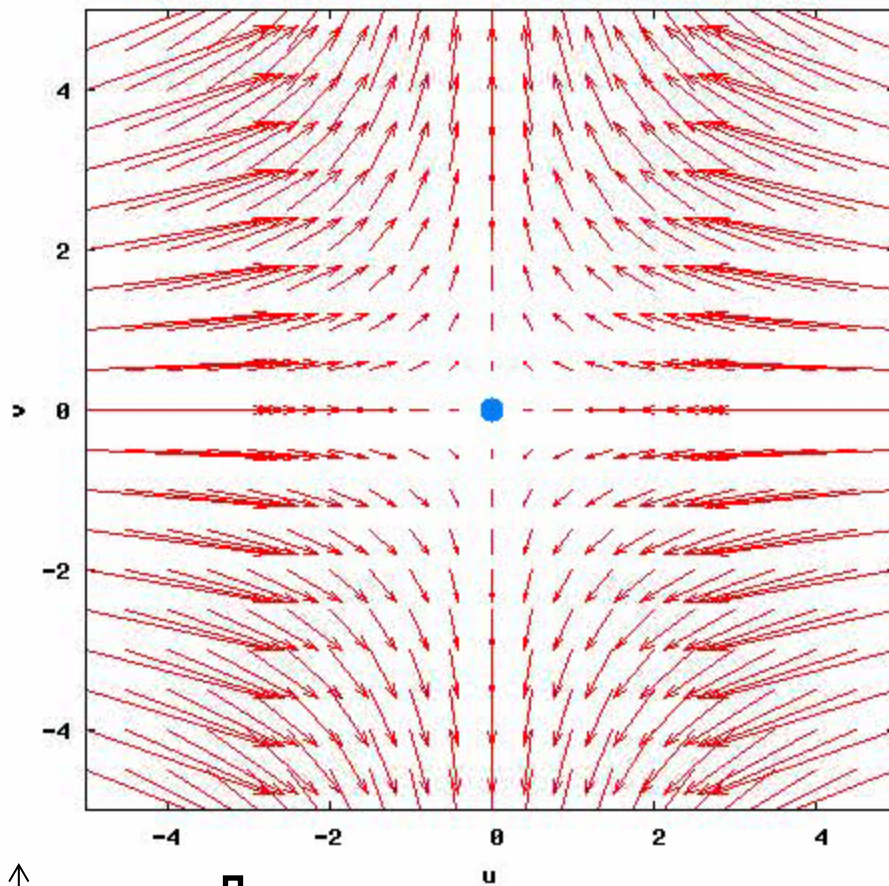
$$\mathbf{A} = \begin{pmatrix} \alpha & 0 \\ 0 & k \end{pmatrix}$$



alpha = -20.00

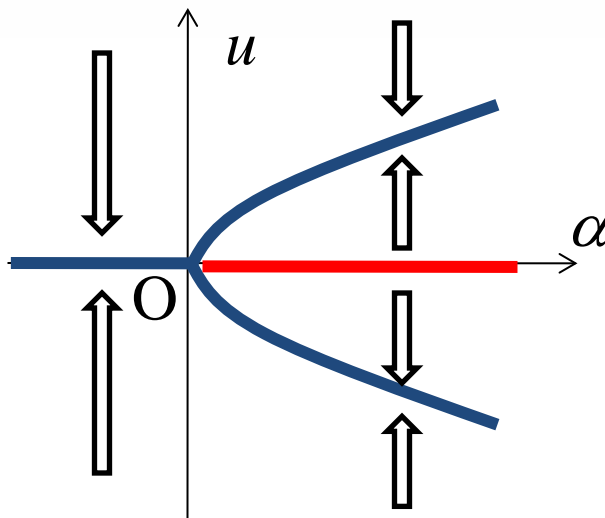


alpha = -20.00



$$\frac{du}{dt} = \alpha u - u^3 \quad k = -20$$

$$\frac{dv}{dt} = kv$$



$$k = 20$$

- 安定性交替分岐

$$\frac{du}{dt} = \alpha u - u^2$$

$$\frac{dv}{dt} = kv$$

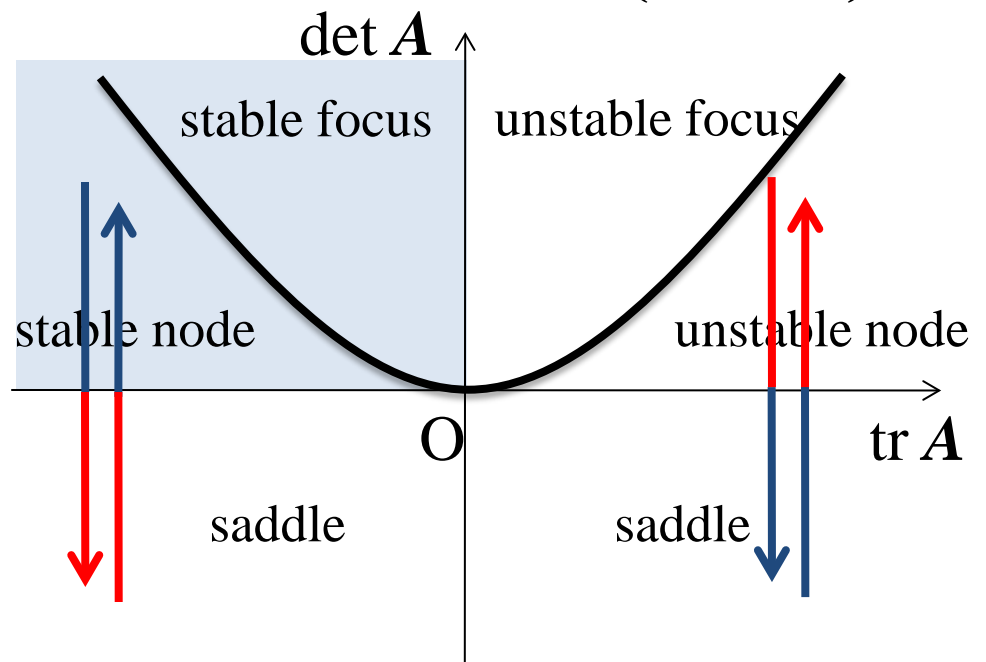
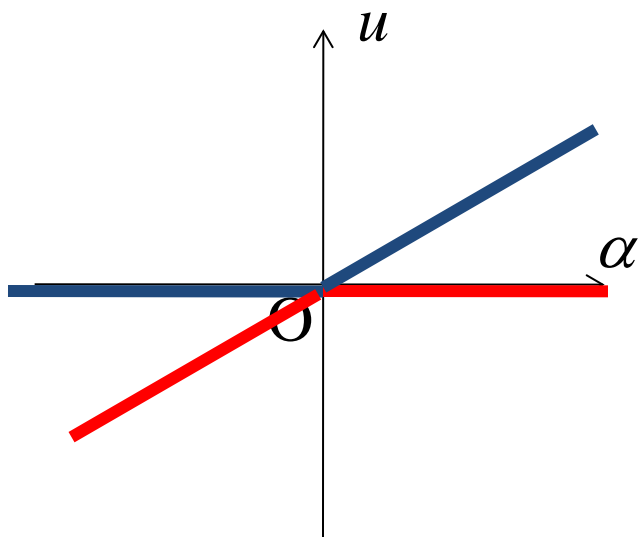
固定点：
 $u=0, \lambda$
 $v=0$

$u=\alpha, v=0$ まわりで

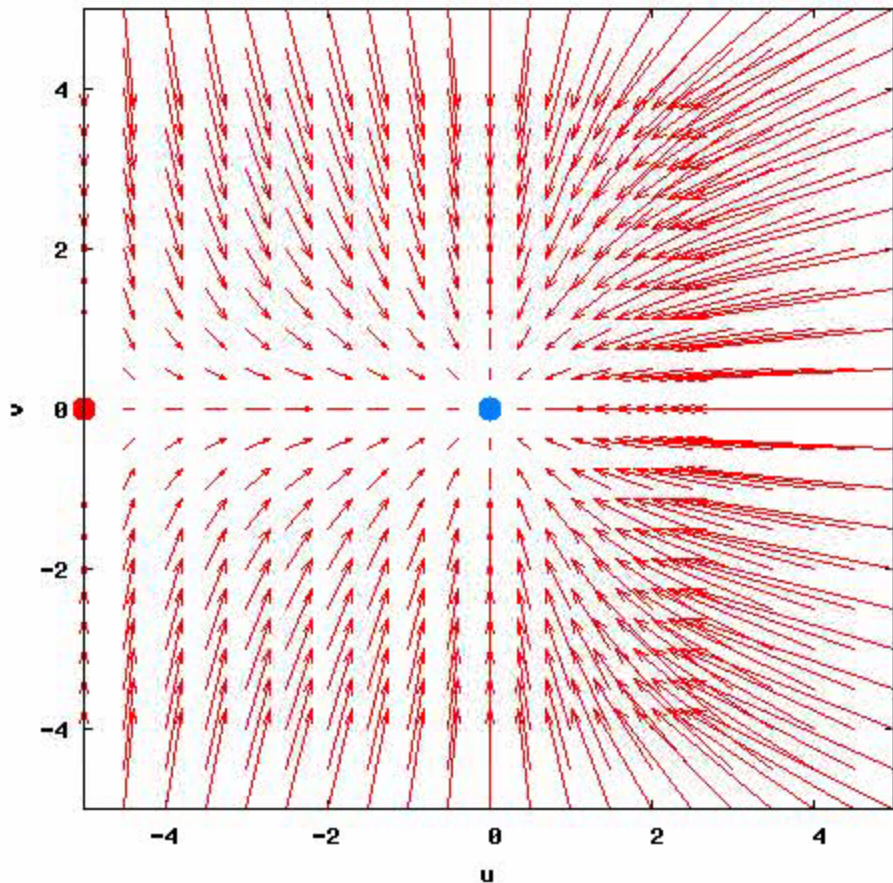
$$\mathbf{A} = \begin{pmatrix} -\alpha & 0 \\ 0 & k \end{pmatrix}$$

$u=0, v=0$ まわりで

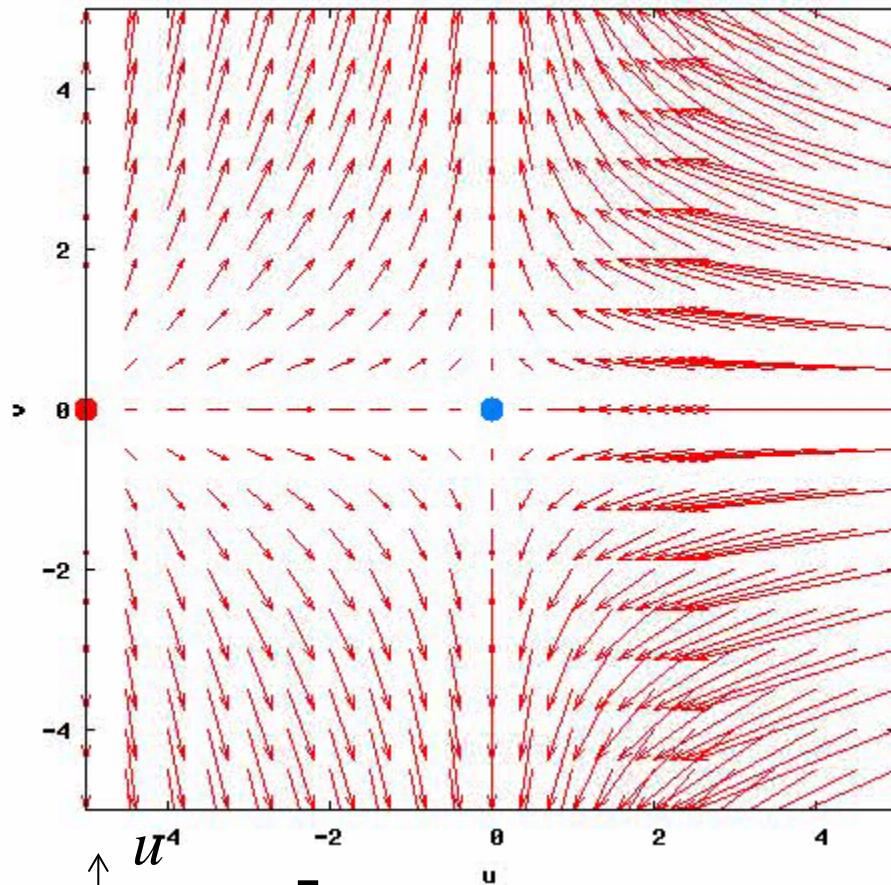
$$\mathbf{A} = \begin{pmatrix} \alpha & 0 \\ 0 & k \end{pmatrix}$$



alpha = -5.00

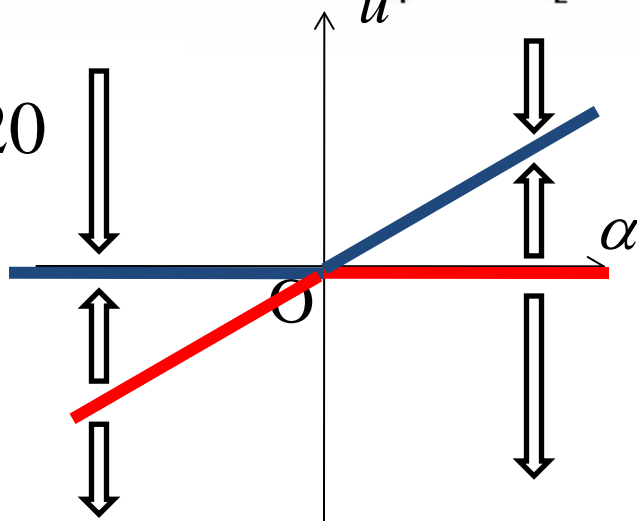


alpha = -5.00



$$\frac{du}{dt} = -\alpha u + u^3 \quad k = -20$$

$$\frac{dv}{dt} = kv$$



$k = 20$

- ホップ分岐

固定点 :

$$u=0 \quad v=0$$

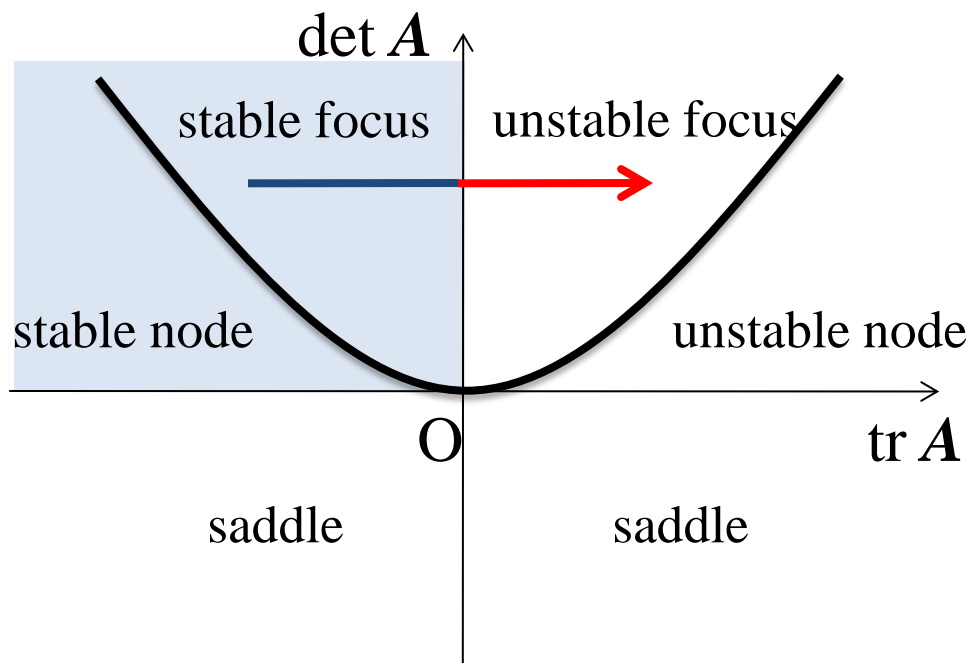
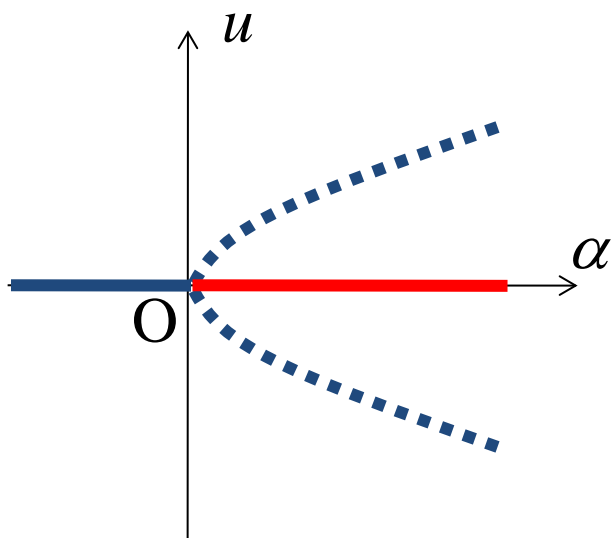
$u=0, v=0$ まわりで

$$\frac{du}{dt} = -v + \alpha u - (u^2 + v^2)(u - \beta v)$$

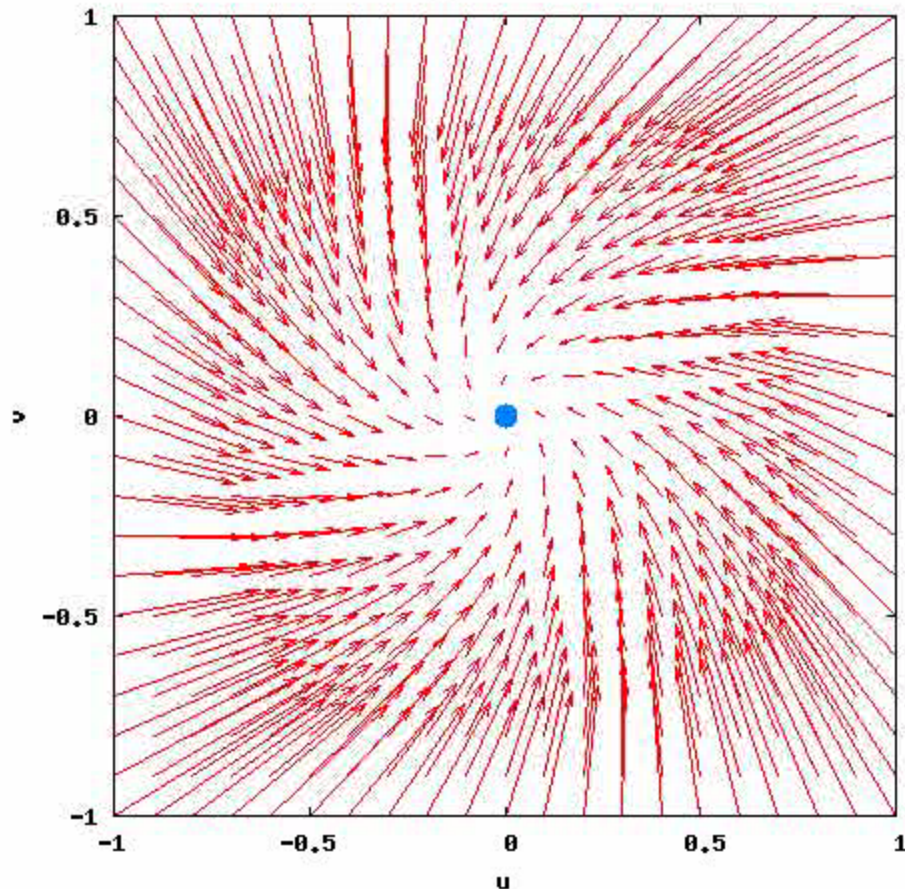
$$\frac{dv}{dt} = u + \alpha v - (u^2 + v^2)(v + \beta u)$$

$$\mathbf{A} = \begin{pmatrix} \alpha & -1 \\ 1 & \alpha \end{pmatrix}$$

$$\lambda = \alpha \pm i$$



alpha = -2.00



$\beta = 0$

$$\frac{du}{dt} = -v + \alpha u - (u^2 + v^2)(u - \beta v)$$
$$\frac{dv}{dt} = u + \alpha v - (u^2 + v^2)(v + \beta u)$$

